Polarization Diversity for Overlapping Cellular Networks
Diversité de Polarisation pour les Réseaux Cellulaires Co-localisés

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This paper investigates the impact of the polarization on the link probability of transmission and the throughput in the context of the cellular network architectures. A channel link model is proposed in the specific case of dual-polarized, Rayleigh-faded communication links. The per-node throughput is analyzed and the gain achieved by using two distinct polarization modes is discussed. It appears that, for uniform random placement of the nodes in a scenario where two cellular networks are deployed on a same location, the dual-polarized channel is an efficient diversity technique.

Dans ce papier, nous nous intéressons à l'évaluation du débit d'un réseau cellulaire bi-polaire. Afin de décrire de manière généralisée l'impact de la polarisation sur les performances d'un réseau, nous dérivons une expression complète de la probabilité de transmission sur un lien donné, dans le cas d'une transmission bi-polaire et sujette à un évanouissement de Rayleigh. Le débit sur un lien donné est analysé et le gain obtenu en utilisant deux modes de polarisation distincts mis en évidence. Nos résultats montrent que, pour un placement régulier des noeuds dans un scénario où deux réseaux cellulaires sont co-localisés, un canal de communication bi-polaire offre une diversité importante.

Key Words – Mots-clefs: Wireless networks, polarization, channel modeling, diversity techniques – *réseaux sans-fil, polarisation, modélisation du canal, technique de divesité*

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1. INTRODUCTION

The 4G systems are expected to support important data rates in the presence of an increasing number of user mobile terminals. Even though the polarization diversity is not a recent concept [1, 2], it has been under-utilized for several years and appears as one of the interesting means to further increase the throughput of the cellular networks [3, 4]. More specifically, in the context of the *cognitive network* architecture [5] where multiple systems overlap, the polarization diversity will allow the joint operation of a primary and a secondary network deployed in a same location.

We address in this paper the throughput model for dual-polarized cellular networks. To provide insight on the impact of the polarization strategy on the network performance, we derive a *closed-form expression* of the probability of transmission on a single link and per time slot, with respect to the topology and the characteristics of the other mobile terminals. The throughput is a performance measure and an indication of the possible kind of traffic that the network will support (e.g., high-rate voice packets or best-effort data packets). We consider slotted ALOHA, which is a simple random access scheme often used and for which, in each timeslot, every node transmit with a fixed probability Λ (heavy traffic hypothesis). Even though ALOHA is a simpler model than TDMA or FDMA, it provides a lower bound on the performance for more elaborate schemes. The remainder

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of this article is organized as follows. In Section 2, the dual-polarized channel model is introduced and a closed-form expression for lin probability of success is detailed. Section 3 focuses on the performance analyses in the context of co-located cellular networks. The section begins with the definition of the analytical throughput for a random access channel. Two distinct scenarios are considered : (i) the co-existence of two WiFi systems and (ii) the co-existence of two GSM/UMTS systems. Also, the impact of the distance on the gain achieved by the diversity of polarization is investigated. The Section 5 concludes the paper.

2. LINK PROBABILITY OF SUCCESS

2.1 The Dual-Polarized Wireless Network

The wireless channel is modeled as a Rayleigh fading channel, i.e., the power at the receiver is [8]

$$P_r = P d^{-\alpha} |\mathcal{R}|^2 \tag{1}$$

where P is the transmitted power, $|\mathcal{R}|$ is a Rayleigh-distributed random variable with parameter σ^2, α is the path loss exponent, and d is the link distance. Therefore, the random variable $X \sim |\mathcal{R}|^2$ has a gamma distribution with parameters 1 and $2\sigma^2$, i.e., $X \sim \Gamma(1, 2\sigma^2)$, and its probability density function is

$$p_X(x) = \frac{1}{2\sigma^2} \exp\left(-\frac{x}{2\sigma^2}\right) U(x)$$

where $U(x)$ is the unit step function and $2\sigma^2 = P_r$, the average received power. A dual-polarized wireless channel uses two distinct polarization modes referred to as *co-polar* (symbol : $\|$) and *cross-polar* (symbol : $\perp$), respectively. In the following, the notions of co- and cross-polar will be used with respect to the transmission of interest. As then, a co-polar interferer will refer to an equipment emitting in the same polarization state than the wireless link of interest. The relation (1) can be written for a transmission on the co-polar and the cross-polar channels, as :

$$P_r^{\|} = P^{\|} d^{-\alpha} |\mathcal{R}|^2 \tag{2}$$

$$P_r^{\perp} = P^{\perp} d^{-\alpha} |\mathcal{R}|^2 \tag{3}$$

Although the two modes are significantly distinct at the emission, the de-polarization increases with the distance so that, at some point, a signal sent on a given polarization channel “leaks” into the other channel. These leaked powers are also subject to Rayleigh fading and the corresponding power values at receiver will be modeled as

$$P_r^{\|\rightarrow\perp}(d) = P^{\|\rightarrow\perp} d^{-\alpha} |\mathcal{R}|^2 \tag{4}$$

$$P_r^{\perp\rightarrow\|}(d) = P^{\perp\rightarrow\|} d^{-\alpha} |\mathcal{R}|^2 \tag{5}$$

where the notations $\| \rightarrow\perp$ and $\perp\rightarrow \|$ represent the leakage from the co-polar to the cross-polar channel and from the cross-polar to the co-polar channels, respectively.

A convenient way to quantify the leakage from one channel to the other is the *cross-polar discrimination* (XPD) coefficient [4]. The XPD is defined as the ratio between the average value power in the emitted polarization and the average value of the power that has leaked to the other polarization. It can be interpreted as the channel ability to discriminate between the two polarizations. By definition, the XPD of a signal on the co-polar channel is

$$\chi_{\|} = \frac{\mathbb{E}\left[P^{\|}\right]}{\mathbb{E}\left[P^{(\|\rightarrow\perp)}\right]} \geq 1$$

Alternatively, a XPD coefficient can be defined to quantify the average ratio of power that has leaked from the cross-polar channel to the co-polar channel :

$$\chi_{\perp} = \frac{\mathbb{E}\left[P^{\perp}\right]}{\mathbb{E}\left[P^{(\perp\rightarrow\|)}\right]} \geq 1 \tag{6}$$

In the following, the XPD value of interest will refer to the interference leakage from the cross-polar to the co-polar channel (that is $\chi_\perp$) and will be noted χ. Extensive measurements show that the value of the XPD varies with respect to the distance [6] and time [7]. It can be modeled as

$$\chi(d) = \begin{cases} \chi_0 \ d^{-\beta} \ \mathcal{L} & , \ d < \chi_0^{1/\beta} \\ 1 & , \ d \geq \chi_0^{1/\beta} \end{cases} \tag{7}$$

where $\mathcal{L}$ is log-normal random variable with parameters 0 and σ_χ, χ_0 is the XPD value at distance $d = 1$m, β is a decay factor ($0 \leq \beta \leq 1$), and $\chi(d) \leq 1$. Note that the temporal variability of the XPD is lower than the temporal variability of the Rayleigh fading by several order of magnitudes. Therefore, only its average value will be considered, that is $\chi(d) \approx \mathbb{E}[\chi(d)] = \chi_0 \ d^{-\beta}$. Using (6) and (7) in (5) gives the power issued in the cross-polar channel and received in the co-polar channel :

$$P^{(\perp \to \parallel)}(d) \ = \ \frac{P^\perp(d)}{\chi(d)} \ |\mathcal{R}|^2 = \begin{cases} \dfrac{P^\perp d^{-\alpha}}{\chi_0 \ d^{-\beta}} \ |\mathcal{R}|^2 & , \ d < \chi_0^{1/\beta} \\[2ex] P^\perp d^{-\alpha} \ |\mathcal{R}|^2 & , \ d \geq \chi_0^{1/\beta} \end{cases} \tag{8}$$

2.2 The Rayleigh Fading Link Model

We assume a narrowband Rayleigh block fading channel. A transmission from an emitter to a receiver is successful if and only if the signal to noise and interference ratio (SINR) γ is above certain threshold value Θ. This threshold value depends on the receiver's characteristics, the modulation, and coding scheme, among others [8]. The SINR of a single link is then given by

$$\gamma \triangleq \frac{P_r}{N + P_{\text{int}}}. \tag{9}$$

where P_r is the received power, N is the noise power, and P_{int} is the total interference power at the receiver, that is, the sum of the received power from all the undesired transmitters. The probability of success of in given link is expressed as $\mathcal{P}_s \triangleq \mathbb{P}\{\gamma > \Theta\}$. Our analysis is based on the following theorem, derived from [9].

Theorem 1. *In a Rayleigh fading network with slotted ALOHA, where nodes transmit with probability Λ, the success probability of a transmission given a fixed transmitter-receiver distance d_0, $N_{int}^{\parallel}$ co-polar interferers at distances d_i ($i = 1, \ldots, N_{int}^{\parallel}$) transmitting at power $P_i^{\parallel}$, and $N_{int}^{\perp}$ cross-polar interferers at distances d'_j ($j = 1, \ldots, N_{int}^{\perp}$) transmitting at power $P_j^{\perp}$ with a cross-polar discrimination coefficient χ_j is*

$$\mathcal{P}_s = \exp\left(-\frac{\Theta N}{P_0 d^{-\alpha}}\right) \times \prod_{i=1}^{N_{int}^{\parallel}} \left\{ 1 - \frac{\Theta \Lambda}{\dfrac{P_0}{P_i^{\parallel}} \left(\dfrac{d_i}{d_0}\right)^{\alpha} + \Theta} \right\} \times \prod_{j=1}^{N_{int}^{\perp}} \left\{ 1 - \frac{\Theta \Lambda}{\dfrac{\chi_j d_0^{-\beta} \ P_0}{P_j^{\perp}} \left(\dfrac{d'_j}{d_0}\right)^{(\alpha-\beta)} + \Theta} \right\} \tag{10}$$

where P_0 is the transmit power, N is the average power of the background noise, and Θ is the SINR threshold value.

Proof. The cumulated interference power at the receiver is defined as the sum of the interferences coming from the co-polarized interferers *plus* the sum of all cross-polarized leakages of power due to depolarization, i.e. :

$$P_{\text{int}} = \sum_{i=1}^{N_{\text{int}}^{\parallel}} P_i^{\parallel} \Lambda_i + \sum_{j=1}^{N_{\text{int}}^{\perp}} P_j^{(\perp \to \parallel)} \Lambda'_j$$

where $\|$ and $\perp$ symbols denote the co- and the cross-polarized channels, respectively. The traffic variables $\Lambda_i \in \{0,1\}$ is a sequence of iid. Bernoulli random variables with $\mathbb{P}\{\Lambda_i = 1\} = \Lambda$ and $\mathbb{P}\{\Lambda_i = 0\} = 1 - \Lambda$. The link probability of correct reception can be expressed as follows :

$$\begin{aligned}
\mathcal{P}_s &= \mathbb{P}\{\gamma > \Theta\} \\
&= \mathbb{E}_{P_{\text{int}}}\left[\mathbb{P}\{\gamma > \Theta\} | P_{\text{int}}\right] \\
&= \mathbb{E}_{P_r,\Lambda}\left[\exp\left(-\frac{\Theta}{\bar{P}_{r,0}}\left(\sum_{i=1}^{N_{\text{int}}^{\|}} P_i^{\|}\Lambda_i \;+\; \sum_{j=1}^{N_{\text{int}}^{\perp}} P_j^{(\perp\to\|)}\Lambda_j\right)\right)\right] \\
&= \exp\left(\frac{-\Theta N}{P_0 d_0^{-\alpha}}\right)\mathbb{E}_{P_r,\Lambda}\left[\prod_{i=1}^{N_{\text{int}}^{\|}}\exp\left(-\frac{\Theta(P_i^{\|}\Lambda_i)}{P_0 d_0^{-\alpha}}\right)\times\prod_{j=1}^{N_{\text{int}}^{\perp}}\exp\left(-\frac{\Theta(P_j^{(\perp\to\|)}\Lambda_j)}{P_0 d_0^{-\alpha}}\right)\right] \qquad (11)
\end{aligned}$$

The expectation in the middle term of (11) can be expressed as follows :

$$\begin{aligned}
\mathbb{E}_{\Lambda_i,P_i^{\|}}[\ldots] &= \mathbb{P}\{\Lambda_i = 1\}\int_0^{\infty}\exp\left(-\frac{\Theta p_i}{P_0 d_0^{-\alpha}}\right)\times p_{P_i^{\|}}(p_i)\mathrm{d}p_i + \mathbb{P}\{\Lambda_i = 0\} \\
&= 1 - \frac{\Theta\Lambda}{\dfrac{P_0}{P_i^{\|}}\left(\dfrac{d_i}{d_0}\right)^{\alpha} + \Theta} \qquad (12)
\end{aligned}$$

The expectation in the rightmost term of (11) can be expressed in a similar way, by using (8) :

$$\begin{aligned}
\mathbb{E}_{\Lambda_j,P_i^{\perp}}[\ldots] &= \mathbb{P}\{\Lambda_j = 1\}\int_0^{\infty}\exp\left(-\frac{\Theta\; p_j}{P_0 d_0^{-\alpha}}\right)\times p_{P_j^{(\perp\to\|)}}(p_j)\mathrm{d}p_j + \mathbb{P}\{\Lambda_j = 0\} \\
&= 1 - \frac{\Theta\Lambda}{\dfrac{\chi_j d_0^{-\beta}\; P_0}{P_j^{\perp}}\left(\dfrac{d'_j}{d_0}\right)^{(\alpha-\beta)} + \Theta} \qquad (13)
\end{aligned}$$

By using (12) and (13) in (11), one finally obtains the probability of successful transmission written in expression (10). □

Theorem 1 gives insightful informations about the expected performance in a dual-polarized transmission subject to background and inter-node interferences. First, the leftmost term of the expression (10) represents the situation where the throughput is limited by the background noise (usually thermal noise). In large and/or dense networks, the transmission is only limited by the interference and we can focus on the interference and polarization parts (i.e., the two other term of the expression, assuming $N = 0$). The first exponential term can be easily evaluated if $N \neq 0$.

The second and the third terms are related to the interference generated by the surrounding nodes in co- and cross-polarization. It depends on (i) the polarization characteristics of the interfering network, (ii) the traffic statistics, and (iii) the topology of the network. These three factors are now discussed.

3. NUMERICAL ANALYSIS

3.1 Throughput Metric

We define a probabilistic throughput value as the success probability of transmission multiplied by the probability that the transmitter actually transmits Λ, i.e.,

$$\tau^{(\text{full})} \triangleq \Lambda\,\mathcal{P}_s$$

In the case of half-duplex operation (e.g., ad-hoc networks, WiFi systems), this value has also to be multiplied by the probability that the receiver actually listens :

$$\tau^{(\text{half})} \triangleq \Lambda(1-\Lambda)\,\mathcal{P}_s$$

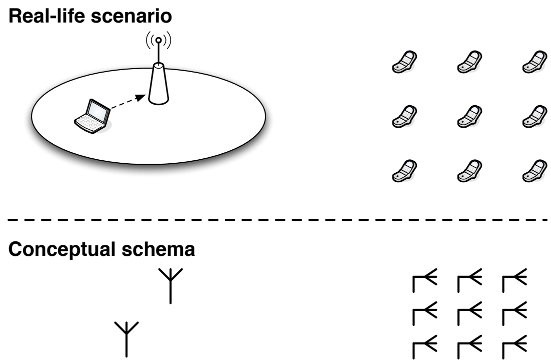


Fig. 1. Generic scenario for the co-existence of primary and secondary WiFi networks.

The probabilistic throughput can be interpreted as the unconditioned reception probability. Finally, the optimal achievable throughput $\tau_{\text{opt}} = \max \tau(\Lambda)$ is obtained for a probability of transmission

$$\Lambda_{\text{opt}} \triangleq \arg\max_{\Lambda} \ \tau(\Lambda)$$

The value Λ_{opt} can be interpreted as the *optimal* packet sending rate (through the probability of transmission) in the sense that it optimizes the throughput on the considered link. In the case of a Poisson-ditsributed traffic, it can be shown [10] that $\Lambda = 1 - \exp(-\lambda L/R_{\text{b}})$ where λ is the average transmission rate (dimension : [b/s]), L is the packet length (dimension : [bit]), and R_{b} is the transmission data-rate (dimension : [b/s]). For that scenario, the optimum packet sending rate is

$$\lambda_{\text{opt}} = -\frac{R_{\text{b}}}{L} \ \ln(1 - \Lambda_{\text{opt}})$$

4. NUMERICAL ANALYSIS

4.1 Scenario

A generic scenario for the co-existence of two overlapping cellular networks is presented in Fig. 3. This scenario is of interest to modelize the co-existence of two different operators in a same place or when a cognitive network approach is used.

In our simulations, the main parameters were fixed with respect to the measurements reported in [6]. More precisely : $\alpha = 2$, $\beta = 0.4$, and $\chi_j = 10$ dB. The SINR threshold value is set to $\Theta = 10$ dB.

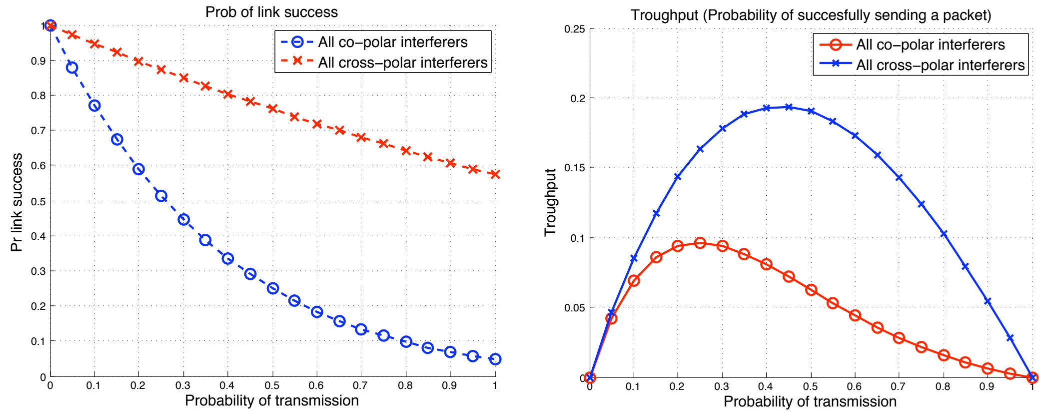
4.2 Scenario 1 – WiFi Networks

In Fig. 1, a topology consisting of a set of 3×3 terminals referred to as the *primary network*. A *secondary network* is deployed as a base station and a connected emitter. Such a situation is typically found in a WiFi architecture or in a small mesh network. The common transmission power is $P = 0$ dBm. The distance d_0 between the terminal and its base station (BS) $d_0 = 20$m. All interferers at $d_i \approx 100$m. In Fig. 2, the probability of link success and the analytical throughput for the secondary network are presented wrt. the two possible polarization states of the primary network.

It can be seen that the use of a dual-polarized approach gives : $\Lambda_{\text{opt}} = 0.45$ for a throughput of $\tau_{\text{opt}} = 0.16$ while, on the other hand, a classical situation (i.e., where all interferers are co-polar) gives : $\Lambda_{\text{opt}} = 0.3$ for a throughput of $\tau_{\text{opt}} = 0.1$. In that scenario, the diversity of polarization allows to double the probabilistic throughput.

4.3 Scenario 2 – Cellular Communications Networks

A single transmission is considered. The cell radius is $r = 2$ km. The distances d_i and d'_j are uniformly distributed over $[0, r]$. The amount of interferers are $N_{\text{int}}^{\parallel} = N_{\text{int}}^{\perp} = 10$. Two different



(a) Probability of link success for the secondary (base station) network.

(b) Analytical throughput.

Fig. 2. Performance analysis for the co-existence of a cellular and a small mesh network.

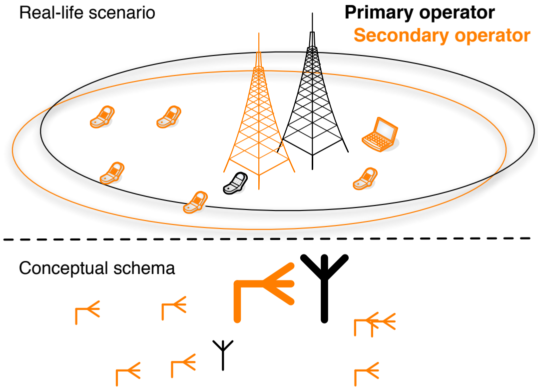


Fig. 3. Generic scenario for the co-existence of two overlapping cellular networks.

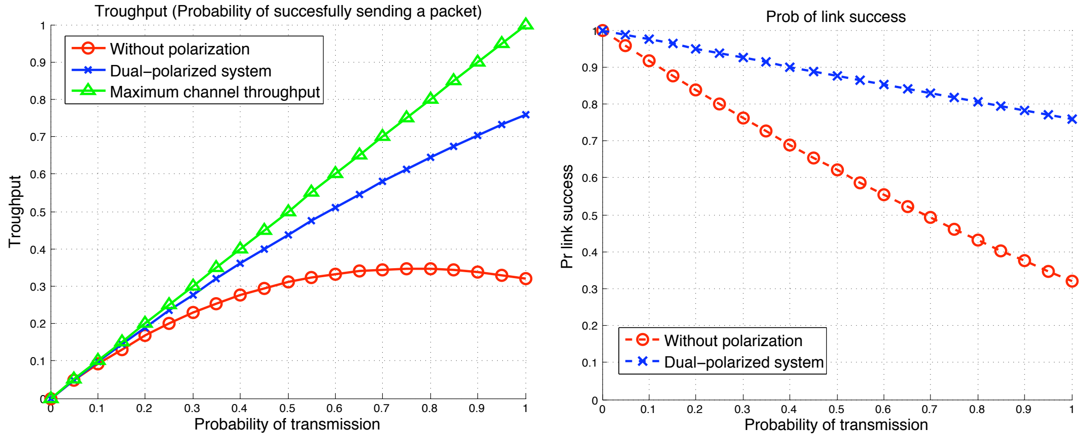
polarization strategies are investigated : (i) the two operators make use of the same polarization (this scenario is referred to as *all co-polar interferers*) and (ii) the operators reduce their interference by using two orthogonal polarization states (this scenario is referred to as *all cross-polar interferers*).

In a first analysis, the distance between the mobile terminal and the base station is set to $d = 100$m. In Fig. 4, the performance is presented, as a function of the probability of transmission and the two polarization strategies. Furthermore, the *maximum channel throughput* is plotted and is defined as

$$\tau_{\max} \triangleq \tau|_{\mathcal{P}_s=1} = \Lambda \tag{14}$$

It can be interpreted ad the maximum theoretically achievable throughput in the wireless channel. It can be seen from Fig. 4 that, for a short transmission distance, the use of polarization yields a throughput gain, especially at high transmission rates. Moreover, the optimal probability of transmission is $\Lambda_{\text{opt}} = 1$ in the case of the dual-polarized network while it saturates to $\Lambda_{\text{opt}} = 0.7$ in the case of the classical approach.

In Fig. 5, the same analysis is conducted for a terminal-to-BS distance of $d = 2$km. It can be seen that the use of a dual-polarized system, is mainly limited by the distance at which both polarization can be discriminated. More precisely, the expression (7) shows that diversity of polarization is beneficial, i.e., $\chi(d) < 1$, if and only if $d < \chi_0^{1/\beta}$. In practice only a fraction of this distance is of interest and the diversity of polarization can be achieved on distances of a few kilometers.



(a) Analytical throughput $\tau^{(\text{full})}$

(b) Probability of link success for a transmission in the primary network, with respect to the probability of transmission and under two polarization strategies.

Fig. 4. Analysis of performance for the two adjacent cellular networks. The mobile to BS distance is set to $d = 100$m.

Finally, we quantify the difference between the cross-polar throughput and the co-polar throughput for a given scenario. This gain value is defined as

$$g = \tau_{\perp}^{(\text{full})} - \tau_{\parallel}^{(\text{full})}$$

where $\tau_{\perp}^{(\text{full})}$ is the throughput obtained for the link of interest is on the cross-polar channel (and the interferers are on the co-polar channel) and $\tau_{\parallel}^{(\text{full})}$ is throughout obtained in the classical approach, i.e., without considering any diversity of polarization.

In Fig. 6, the average value of g is reported as a function of the probability of transmission and with respect to the distance between the mobile and the base station. It can be seen that a dual-polarized deployment allows a significant gain in terms of throughput when (i) the mobile-to-BS distance is small and (ii) when the probability of transmission is high (i.e., in presence of a high packet transmission rate).

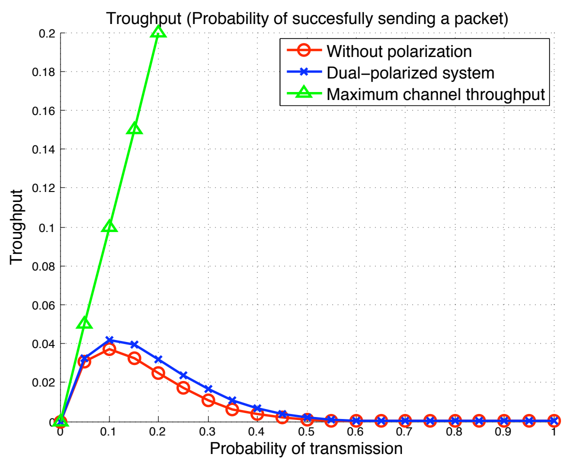


Fig. 5. Analytical throughput $\tau^{(\text{full})}$. The mobile to BS distance is set to $d = 2$km.

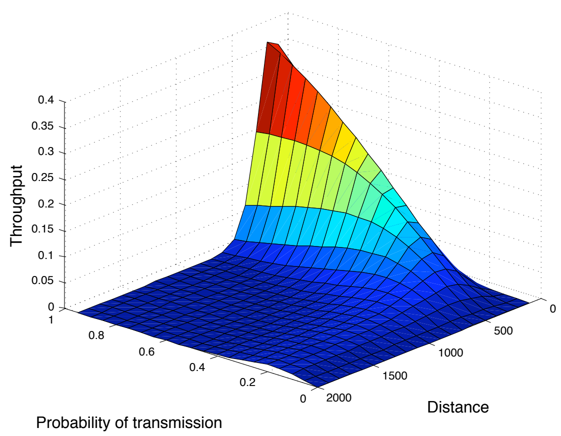


Fig. 6. Difference between the value of throughput on the cross-polar channel and the co-polar channel.

5. CONCLUSIONS

A dual-polarized channel model in the context of the cellular systems was presented and its performance was investigated. We have shown that for noiseless, Rayleigh fading networks, the success probability of a transmission significantly increases when the two networks operate on different polarization states. More precisely, in a scenario where a half-duplex transmission is implemented (i.e., WiFi system), the per-link analytical throughput can be doubled by using the diversity of polarization. In full-duplex cellular networks (i.e., GSM/UMTS systems), the relative gain is less significant at lower data rates but it increases as function of the probability of transmission.

Also, it was noted that in the context of the GSM/UMTS networks, the optimal throughput never saturates when taking advantage of diversity of polarization. This means that, in the dual-poarized channel, any intensification in the packet transmission rate allows to monotonically increase the channel throughput. However, the channel looses the ability to discriminate between the polarization states when the distance between the emitter and the receiver increases. Therefore, diversity of polarization cannot be performant over long distances (e.g., more than a few kilometers).

Finally, even though for cellular networks the MAC schemes in use are more elaborate, our primary analysis provides the lower bounds of the performance for other channel access schemes and demonstrates that the diversity of polarization is a promising technique to increase the offered bandwidth.

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