Transport and turbulence in edge and SOL tokamak plasmas
Transport et turbulence dans le plasma de bord d’un tokamak

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Abstract
In this paper we present the main issues concerning our research activity on tokamak plasmas and numerical simulation. The long term scope of our effort is to develop a 3D simulation code of the edge plasma-wall interaction including turbulent transport. This versatile numerical tool will address the physics in a simplified cylindrical geometry (limiter geometry). This code is also used as a test bed to implement novel numerical schemes that will be successively taken into account by the future (ITER horizon) ESPOIR code for full ITER geometry developed in the framework of the ANR project ESPOIR (ANR-09-BLAN-0035-01, 2009-2013).

Introduction
In this paper we present the main issues concerning our research activity on tokamak plasmas and numerical simulation. The long term scope of our effort is to develop a 3D simulation code for transport and turbulence in edge and SOL tokamak plasmas, able to handle also with plasma-wall interaction. The very sharp transition in the edge-SOL region is a challenge to many concepts used to describe transport properties and requires a powerful numerical treatment able to account simultaneously for the doubly-periodic (inner) region and the essentially non-periodic (outer) region. The difficulty arises from the open magnetic field lines which interact with the limiter, involving speeds for the particles close to the sound speed in the vicinity of the limiter (Fig.1).

Due to the hyperbolic nature of the transport equations in the direction parallel to the magnetic field, shock waves can appear involving strong discontinuities in the fields. The competition between the various stabilizing and destabilizing mechanisms around the last magnetic surface will allow investigating the complex behaviour of the edge plasma including such effects as bifurcation. The operating regime of reference of ITER is a regime of improved confinement (H-mode), which explains the urgency of a theoretical comprehension of this regime as well as its implication on the plasma-wall interaction.

FIG. 1 – Left panel: three-dimensional representation of the tokamak vacuum chamber, with the limiter lying at the bottom of the torus. A magnetic field line is represented in green colour, winding helically around the torus. Right panel: poloidal cross-section of the tokamak. The scrape-off layer corresponds to the area outside the dashed line that denotes the core/SOL transition. In the SOL, parallel flows (flowing along field lines, and therefore poloidally in the cross-section) are driven to...
sound speed (Mach=±1) towards both sides of the limiter. The core plasma is essentially at rest, which implies that significant radial gradients in momentum are expected at the transition between core and SOL, possibly driving instabilities like, for example, a Kelvin-Helmholtz like instability.

SOLEDGE-3D

Our effort is presently dedicated to a versatile simulation tool named SOLEDGE-3D that is able to address the physics in a simplified cylindrical geometry and is also used as a test bed to develop novel numerical schemes further implemented in a code dealing with the full ITER geometry. SOLEDGE-3D is presently running on the high-performance vector computer at the CNRS computational center IDRIS for the momentum and the density transport. Perpendicular motion is retained by including advection of both density $n$ and parallel momentum $\Gamma_{//}$ by the electrostatic drift velocity $\bar{v}_e = \frac{\langle B \times \nabla \phi \rangle}{B^2}$ with $B$ the magnetic field and $\nabla \phi$ the electric field. The system is closed by assuming electron adiabacity, giving the usual relation between the electric potential $\Phi$ and density fluctuations $n$, that is $e\Phi/Te = \ln(n/n_{av})$ where $n_{av}$ is the flux-surface averaged density and $Te$ the electron temperature. Quasi-neutrality assumption (ion density $n_i$ and electron density $n_e$ are such that $n_i = n_e = n$) and electrostatic approximation ($B$ fixed given by MHD equilibrium) as well are considered. Isothermal closure is assumed in the current version ($T_i = T_e = \text{cst}$). We introduce perpendicular diffusion mechanisms both in density and momentum equation with diffusivities $D$ and $\nu$ respectively. Thus we consider the following equations:

$$
\begin{align*}
\partial_t n + \frac{B.(\nabla \phi \times \nabla n)}{B^2} + \nabla_{//} \frac{\Gamma_{//}}{n} - \nabla_\perp \cdot (D \nabla_\perp \phi) &= 0 \\
\partial_t \Gamma_{//} + \nabla_{//} \left( \frac{\Gamma_{//}^2}{n} + n c_s^2 \right) + \frac{B.(\nabla \phi \times \nabla \Gamma_{//})}{B^2} - \nabla_\perp \cdot (\nu \nabla_\perp \Gamma_{//}) &= 0 .
\end{align*}
$$

(1)

Appropriate boundary conditions are implemented with Bohm condition at the limiter boundaries. The code uses Fourier decomposition in the parallel direction while a two order in time and space finite difference method is implemented for the perpendicular direction. The computational domain contains both the core, with periodicity in the poloidal and toroidal directions, and the Scrape-Off Layer where field lines intercept the limiter losing the poloidal periodicity. A field-aligned discretization has been already developed since there is a clear distinction between the direction along the magnetic field and across it. An example of a simulation result for plasma density is given below (decaying mode with toroidal wavenumber 9)

![Fig. 2: Decaying mode with toroidal wavenumber 9 in a 3D simulation](image)

From a physical point of view, the strong variation of the parallel component of the plasma velocity while crossing the last closed magnetic surface, can involve a flow destabilization...
with appearance of Kelvin-Helmholtz like instability and a transport of the momentum in the parallel direction. The features of Kelvin-Helmholtz instability have been currently studied, in a cylindrical computational domain (without limiter) using a linear stability analysis. Equilibrium maps (with radial and poloidal variations) have been started to be investigated in the geometry with limiter for large values of particle diffusion and viscosity, which in the absence of turbulence mechanism determine the radial gradients at the transition between the core plasma and the SOL. Such purely diffusive equilibriums require large 3D computations with long time to reach the steady state. They should show radial gradients of the parallel flow at the transition exceeding the linear instability threshold. These numerical solutions will be analyzed globally in the Kelvin-Helmholtz linear instability framework with profiles obtained for different values of the diffusion coefficients.

**Penalization technique for modelling plasma-obstacle interaction**

Another relevant issue we recently addressed is the modelling of limiters and antennas using an original penalization technique. At present, this technique has been implemented in the 1D and 2D version of the code SOLEDGE. Its extension to the 3D case is under way.

Contrary to a neutral fluid, for charged plasma the interaction with a solid object means the absorption of the plasma by the solid. For this reason limiters, or antennas, are considered as pure particle sinks for the plasma and consequently the density and the momentum are enforced to be zero inside these solid objects. Usually, limiter or divertor geometries are taken into account in numerical code by implementing the Bohm boundary conditions on the obstacle surface. Due to the very fast dynamics of the plasma at the limiter interface, such boundary conditions on the surface of the obstacle can require body-fitted unstructured meshes or remeshing strategies which can be time consuming. In our approach, the plasma-limiter interaction is no longer described by demanding to density and velocity to satisfy Bohm conditions.

The limiter (or obstacle) \( \Omega \) is represented by a characteristic function \( \chi \) satisfying \( \chi = 1 \) in the spatial region of the limiter and zero in the plasma domain. Density and momentum depend now on the penalization parameter \( \eta \) which must be small in the following system which corresponds to the 2 dimensional version of system (1):

\[
\partial_t n + \nabla \cdot (\Gamma_{\parallel}n + \frac{\chi}{\eta}(n-0)) = \partial_r (D \partial_r n),
\]

\[
\partial_t \Gamma_{\parallel} + \nabla \cdot \left( \Gamma_{\parallel} \left( \frac{\Gamma_{\parallel}^2}{n} + nc_s^2 \right) + \frac{\chi}{\eta}(\Gamma_{\parallel} - \Gamma_{\Omega}) \right) = \partial_r (\nabla \cdot \Gamma_{\parallel}).
\]

The penalization issues have been addressed by analyzing a one-dimensional nonlinear hyperbolic problem corresponding to the rapid transport in the parallel direction. In that way, this numerical work extends former mathematical studies restricted to linear equations. The penalization error corresponding to the difference between the solution of the penalized equations and the exact solution of the equations with the surface Bohm boundary condition can be controlled by choosing appropriate values of the penalization parameter \( \eta \). As reported in the literature, the local error defined inside of the limiter is shown to decay linearly with \( \eta \) that gives a real control on the way the boundary conditions are imposed numerically. Since the periodicity of the domain is recovered, present results have also shown the suitability of Fourier spectral methods in spite of a relatively poor convergence result of the truncation error for this method due to Gibbs oscillations. A very attractive feature of the
present method is that it provides a plasma velocity which is almost sonic at the boundaries obstacles as expected from the sheath conditions through the Bohm criterion.

Moreover, two-dimensional simulations have shown that the physics of the SOL is recovered, in particular regarding the radial profile of the density averaged in the parallel direction. This approach also allows one to readily take into account the radial diffusive transport of particles to the limiter head. This is particularly important when the limiter has a finite parallel extension. A consequence of this new physics is to slightly modify the flow pattern, in particular by enforcing a flow in the edge plasma to compensate for this specific sink. As a consequence, the total plasma pressure variation, which characterizes the diffusive momentum transfer, is slightly modified although the overall shape is unaffected. It exhibits a momentum source region localised in the edge and SOL plasma near the separatrix coupled to momentum sinks both in the plasma core and in the outer SOL. Since the total pressure is larger in the edge plasma than in the outer SOL, the momentum spreading is more effective toward the outer SOL.

Having validated the penalization method for the plasma/limiter interaction, it is straightforward to address the multi-limiter case. We have considered geometry with a primary limiter together with a secondary limiter typically at 2 e-folding lengths in the far SOL. This simulation has allowed us to analyse the properties of the secondary SOL and in particular its small e-folding length. One has also found that the plasma flow pattern was modified, including the flow pattern in the edge plasma. One therefore finds that the transport properties are changed by the secondary limiter.

The complexity introduced by the Bohm boundary condition for the plasma-wall interaction has strongly restrained the simulations of the edge and SOL plasmas. The proposed penalization technique (see ref. [1]) alleviates these difficulties and allows one to implement in a rather straightforward fashion realistic plasma facing components. Of course, introducing small objects will have implication in the required code resolution. However, for a code based on a proper description of the turbulent transport, hence with a typical mesh size given by the ion Larmor radius; this issue will be addressed anyhow. As shown by the few results proposed in this paper, the capability to properly describe these components can affect the overall transport pattern up to the edge plasma. The penalization technique thus appears as a powerful tool to investigate the SOL and edge plasma transport properties, and in particular assess the geometrical effects first addressed in the examples chosen to illustrate the present work.

This work is performed in the framework of the project ANR ESPOIR (Edge Simulation of the Physics Of Iter Relevant Turbulent Transport, 2009-2013), dedicated to global 3D simulations of plasma-wall interaction including the key turbulent media. This...
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