A numerical simulation for brain stroke microwave imaging by using the FVTD

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Keywords: medical imaging, inverse scattering, microwaves, numerical methods.

Introduction

Brain stroke is a severe and widespread pathology that involves a significant percentage of the population (e.g., 1.7% of the general population and 10-15% of people over 75 years old in Italy). A stroke occurs when the blood flow inside the brain is interrupted by a blocked (ischemic stroke) or burst (hemorrhagic stroke) blood vessel. As a consequence, brain cells begin to die and the abilities controlled by that part of the brain can be lost. An efficient and fast diagnosis able to discriminate between the two types of stroke is essential for an immediate and effective therapy [1].

Among the various currently proposed imaging modalities, the use of microwaves has been reported in some recent studies (see, for example, [2], [3] and the references therein.) It is well known that microwave-based imaging techniques are widely considered in several fields in which one has to probing unknown matter. A significant example is represented by the area of nondestructive testing and evaluations (NDT&E). In such approaches, the target is illuminated by means of low-power non-ionizing microwaves and the scattered electric field is collected outside the target by using one or more receiving antennas. Information about the materials of the target can be obtained by inverting the equations of the inverse scattering problem (which can be formulated in several different ways.)

Among the various proposed approaches, the present authors wish to exploit the use of a deterministic technique based on an inexact-Newton method, which has been found to be effective in NDT&E areas [4]–[9]. However, before considering the inversion of data, it is mandatory to exploit the response of the imaging system in terms of dynamic range and scattered field levels. Therefore, in the present paper, the results of a series of numerical simulations are reported in order to obtain some guidelines concerning the imaging apparatus as well as to devise suitable operating imaging conditions.

Another key point of the paper is the use of a recently proposed numerical method, i.e., the Finite-Volume Time-Domain (FVTD) method, which is applied in order to analyze in the time domain the interaction between the incident illuminating wave and the brain stroke, through the use of an anatomically realistic phantom head. The FVTD method seems to be very effective in solving Maxwell’s equations in several electromagnetic scenarios, since it ensures both a high geometrical flexibility compared to other time-domain methods and the possibility of modeling complex structures including small feature and curved surfaces [10].

1. Finite-Volume Time-Domain method

A classical microwave tomographic configuration is shown in Fig. 1. Transverse magnetic illumination conditions (with respect to the \( \mathbf{z} \) axis) are assumed. A cylindrical target (corresponding to a slice of a realistic model) is positioned with its cross section inside an investigation area. The dielectric properties of the target are assumed to be independent from the coordinate \( z \). Under such hypothesis, the scattering problem can be described in terms of two-dimensional and scalar equations. The transmitting antenna is modeled as a line current source positioned at \( \mathbf{r}_x \) and the electric field is evaluated in a set of \( M \) points located outside the head phantom, at points \( \mathbf{r}_m \), \( m = 1, ..., M \). The Finite-Volume Time-Domain method is used to solve the direct scattering problem by computing the electromagnetic fields.

Let us consider the investigation area \( D \) partitioned into \( N \) elementary subdomains \( D_i \) such that \( D = \bigcup_{i=1}^{N} D_i \). Under these assumptions, the following balance law over each element \( D_i \) can be written from the Maxwell’s equations [10], [11]

\[
\frac{d}{dt} \int_{D_i} U(\mathbf{r}, t) dS = -\alpha_i^{-1} \int_{\partial D_i} \mathbf{F}(U)(\mathbf{r}, t) \cdot \mathbf{n}_i dI - \alpha_i^{-1} K_i \int_{D_i} U(\mathbf{r}, t) dS
\]

(1)
where \( \mathbf{n}_i = n_{ix}\mathbf{i} + n_{iy}\mathbf{j} \) denotes the outward normal vector to the boundary \( \partial D_i \) of \( D_i \), \( \mathbf{U} = (E_z, H_x, H_y)^T \) (where the superscript \( T \) indicates the transpose operator), and

\[
\mathbf{F}(\mathbf{U})(\mathbf{r}, t) \cdot \mathbf{n}_i = \begin{pmatrix}
-H_y(\mathbf{r}, t) & H_x(\mathbf{r}, t) \\
0 & E_z(\mathbf{r}, t)
\end{pmatrix} \begin{pmatrix}
n_{ix} \\
n_{iy}
\end{pmatrix}
\]

\( \alpha_i = \begin{pmatrix}
\varepsilon_i & 0 & 0 \\
0 & \mu_i & 0 \\
0 & 0 & \mu_i
\end{pmatrix}
\)

\( K_i = \begin{pmatrix}
\sigma_i & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{pmatrix}
\)

being \( \varepsilon_i, \mu_i \) and \( \sigma_i \) the dielectric permittivity, the magnetic permeability, and the electric conductivity of \( D_i \) respectively. The considered FVTD formulation is based on the computation of the values of the electric and magnetic fields at the center of each element \( D_i \). Consequently, the method evaluates, for \( i = 1, \ldots, N \), the quantities

\[
\mathbf{U}_i^n = \mathbf{U}(\mathbf{r}_i, n\Delta t) \equiv \frac{1}{|D_i|} \int_{D_i} \mathbf{U}(\mathbf{r}, n\Delta t) dS, \quad n = 0, \ldots, N_t
\]

being \( |D_i| \) the area of the element \( D_i \), \( \mathbf{r}_i \) the center of the \( i \)th subdomain, and \( \Delta t \) the time step. In order to deal with materials characterized by non-negligible electric conductivity, the Fractional-Step Technique (FST) [11] is adopted. In such an approach, the problem is split in two sub-problems described by the following set of equations

\[
\frac{d}{dt} \int_{\partial D_i} \mathbf{U}(\mathbf{r}, t) dS = -\alpha_i^{-1} \int_{\partial D_i} \mathbf{F}(\mathbf{U})(\mathbf{r}, t) \cdot \mathbf{n}_i dl
\]

\[
\frac{d}{dt} \int_{\partial D_i} \mathbf{U}(\mathbf{r}, t) dS = -\alpha_i^{-1} K_i \int_{\partial D_i} \mathbf{U}(\mathbf{r}, t) dS
\]

The first one refers to the computation of the fields in lossless dielectrics, whereas the second one is an ordinary differential equation taking into account the effects of the non-zero conductivity and whose solution is given by [11].

\[
\int_{\partial D_i} \mathbf{U}(\mathbf{r}, t) dS = e^{-\alpha_i^{-1} K_i(t-t_0)} \int_{\partial D_i} \mathbf{U}(\mathbf{r}, t_0) dS
\]
By considering both a flux-splitting formulation and the Monotone Upstream-centered Schemes for Conservative Laws (MUSCL) [10], and by applying, as a time marching scheme, the second order Lax-Wendroff predictor-corrector criterion, the explicit update equations at the nth temporal step are given by [10], [11]

\[
U_i^{n+0.5} = U_i^n - \alpha_i^{-1} \frac{\Delta t}{2|D_i|} \sum_{k=1}^{3} |t_i^k| \left( \alpha_i T_i \Phi_{i,k}^{+,n} + \alpha_j T_j \Phi_{j,k}^{-,n} \right)
\]

\[
U_i^{n+1} = e^{-\alpha_i^{-1} \kappa_i |t_i|} \left( U_i^n - \alpha_i^{-1} \frac{\Delta t}{|D_i|} \sum_{k=1}^{3} |t_i^k| \left( \alpha_i T_i \Phi_{i,k}^{+,n+0.5} + \alpha_j T_j \Phi_{j,k}^{-,n-0.5} \right) \right)
\]

2. Numerical results
A wide numerical analysis has been performed in order to evaluate the response of the head model. Firstly, we studied the effects of different matching media on the penetrated field; secondly, we analyzed the electromagnetic field response in the presence of two typologies of simulated stroke: a hemorrhagic one and an ischemic one.

In order to perform our simulations, a realistic human head phantom has been considered. This phantom derived from the Zubal model [12], which represents an anatomically realistic human head model obtained from Magnetic Resonance images of a living male and consists of 256 × 256 × 128 voxels of sides 1.1 mm × 1.1 mm × 1.4 mm. Two different slices have been considered in order to take into account different tissues configurations: the first one located at 56 mm from the top; the second one, more complex, located at 86.8 mm from the top. The incident electric field is generated by a line current source located at \( r_0 = -11.5 \) cm and is excited by a modulated Gaussian pulse with a bandwidth of 1 GHz centered at \( f_0 = 1 \) GHz. The electric field has been evaluated in 5 inner points, i.e., positioned at \( r_{\text{int}} = 0 \), \( r_{\text{int}}^2 = 2.5 \) cm, \( r_{\text{int}}^3 = -2.5 \) cm, \( r_{\text{int}}^4 = 5 \) cm, and \( r_{\text{int}}^5 = -5 \) cm.

The considered computational domain is a square region \( D = [-L/2,L/2] \times [-L/2,L/2] \), being \( L = 28.05 \) cm, which has been uniformly partitioned into 32768 right triangles having both catheti of length \( 2.2 \) mm. In order to suppress numerical reflections at the boundaries of the computational domain, first order Silver Muller’s absorbing conditions were applied.

The dielectric properties of the brain tissues were taken from [13] and different matching media, ranging from air (\( \varepsilon_r = 1 \)) to water (\( \varepsilon_r = 80 \)) have been considered for the evaluation of the electric field penetration inside brain tissues. In particular, the effects on the penetrated electric field have been evaluated by considering the following parameter

\[
e = \frac{\| E_{\text{tot}}(r_{\text{int}}, t) - E_{\text{inc}}(r_{\text{int}}, t) \|_2}{\| E_{\text{inc}}(r_{\text{int}}, t) \|_2}
\]

(11)

where \( E_{\text{tot}} \) and \( E_{\text{inc}} \) are the electric fields computed with and without the head and \( \| \cdot \|_2 \) the \( l^2 \) norm. From our analysis, we obtained the best coupling by considering a material with relative dielectric permittivity \( \varepsilon_r^{HM} \) in the range between 10 and 20.

Subsequently, the response of two different kind of brain stroke was analyzed by considering both a hemorrhagic and an ischemic stroke. The first one arises from bleeding and its dielectric properties are equal to those of the blood (i.e., relative dielectric permittivity \( \varepsilon_r^{HS} = 61 \) and electric conductivity \( \sigma^{HS} = 1.5829 \) S/m), while the second arises from the obstruction of a blood vessel, thus causing a thrombus, so as its dielectric properties are considered to be those of a clot (i.e., relative dielectric permittivity \( \varepsilon_r^{IS} = 36 \) and electric conductivity \( \sigma^{IS} = 0.72 \) S/m) [2]. Both the anomalies have been modeled as an elliptically-shaped inclusion with center \( C_0 = 2.5 \) cm and different dimensions.

The effects of the presence of both the hemorrhagic and the ischemic stroke on the scattered electric field have been assessed by evaluating the quantity

\[
e_{\text{pert}}(r_m, t) = \frac{E_{\text{stroke}}(r_m, t) - E_{\text{no stroke}}(r_m, t)}{\| E_{\text{inc}}(r_m, t) \|_2}
\]

(12)

where \( E_{\text{stroke}} \) and \( E_{\text{no stroke}} \) are respectively, the electric fields (\( z \)-component) computed with and without the stroke and \( E_{\text{inc}} \) is the electric field computed without the head. The measurement points \( r_m \) have been chosen equally spaced and uniformly distributed on a circumference of radius \( R = 11.50 \) cm. An example of the obtained results is reported in Fig. 2(a) and Fig. 2(b) and the assumed data for this simulation are provided in the caption.
Fig. 2. (a) Cross section of the head model, at 86 mm from the top, with the simulated hemorrhagic stroke. (b) Behavior of the parameter \( \varepsilon_{\text{pert}} \) for different measurement points, by considering the slice at 86 mm from the top with a matching medium of \( k = 10 \). The hemorrhagic stroke has major axis \( z = 4 \) cm and minor axis \( b = a/2 \). The source is located at \( r_5 = -11.5 \) cm.

Citations


