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A new polynomial-chaos based inversion algorithm

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Abstract

Due to the high computational cost of physical models, metamodels (also known as surrogate models) have gradually appeared as crucial alternatives. Polynomial chaos (PC) expansions are a type of metamodel based on an explicit spectral decomposition of the physical model, allowing for an exact computation of the gradient of the metamodel. In the context of inverse scattering problems, the gradient of the metamodel is used to improve the performance of the global algorithm Particle Swarm Optimization (PSO).

1 Introduction

Inverse scattering problems have many applications, ranging from medical imaging to nondestructive testing. They can be thought as an optimization problem, and can be solved using iterative methods which aim at progressively refining the estimation of the parameters thanks to a large number of physical model evaluations. However, physical models are often computationally expensive to evaluate. To avoid the computational expensiveness of exact models, surrogate models - or metamodels - can be used. A metamodel is an approximation of the explicit model, which provides a much faster model evaluation with a small loss in precision due to some approximations. In this work, the potential complementarity between stochastic global optimization and deterministic local optimization is investigated, provided the gradient of the metamodel is computable.

2 Theoretical framework

This algorithm relies on two steps: first, building the metamodel between input and output domains, finally integrating this metamodel inside an iterative optimization algorithm to retrieve the sought parameters.

2.1 Metamodeling: polynomial chaos

Polynomial chaos (PC) is a widely used technique which, most often, finds its utility in uncertainty quantification (UQ) problems [1], but can also be used as metamodel [2]. PC expansions rely on a spectral decomposition on a basis of orthogonal polynomial functions. The coefficients of the expansion are computed thanks to the *UQLab* toolbox [3] and the polynomial basis is pre-determined by the input distribution. One significant advantage of the PC expansion is its exact analytical expression, from which the exact gradient can simply be extracted [4].

2.2 Optimization: particle swarm optimization

First introduced in [5], then thoroughly discussed in [6], Particle Swarm Optimization (PSO) is an iterative optimization algorithm. Two optimization strategies are proposed: the first one is solely based on the PSO algorithm, the second one uses the gradient information. Both strategies use the same cost function. The second strategy, referred to as gPSO, also uses the gradient of the cost function to provide a deterministic way to displace the swarm inside the search space. This is expected to be effective in case the cost function is convex for instance.

3 Numerical results

The algorithm has been applied to a NdT configuration. A plate structure affected by a rectangular crack is inspected by a coil. The problem has 3 parameters, which form the input domain. The output domain is a C-scan of the plate, which is a complex-valued rectangular image. The dimension of the output domain is relatively high and thus can be reduced to improve computation times. To achieve this, a principal component analysis [7] is used.

The simulations have been carried out on 100 points sampled by Latin Hypercube Sampling in the input domain. The algorithm stops when the cost function value reaches 10^{-2} or when 150 iterations have passed. The results are shown in Figure 1 for the first strategy, and Figure 2 for the second one as scatterplots of reconstructed value versus true value for each parameter. In the case of a perfect reconstruction, the expected result is a straight line of equation $y = x$. The average number of iterations over these 100 points is 47.25 for the PSO and 8.69 for the gPSO.

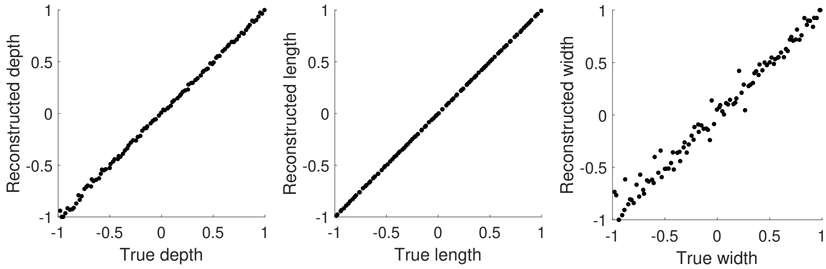

Figure 1: Input parameter reconstruction by PSO: depth (left), length (middle) and width (right) on 100 points sampled by LHS

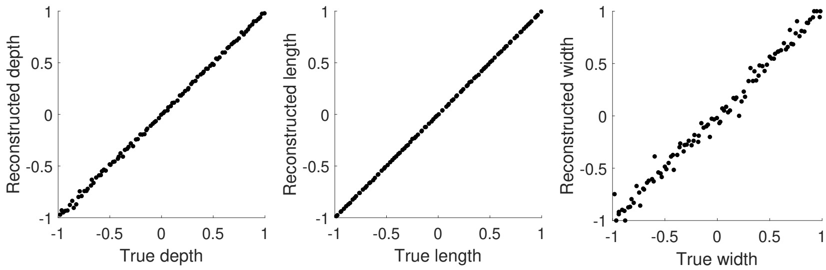

Figure 2: Input parameter reconstruction by gPSO: depth (left), length (middle) and width (right) on 100 points sampled by LHS

4 Conclusion and future work

An original inversion algorithm based on polynomial chaos and PSO has been introduced and tested on an eddy-current testing configuration. The combination of local optimization from the gradient descent algorithm and global optimization from PSO improves the convergence speed. The algorithm is expected to be tested on other configurations, with an increasing number of parameters, and its robustness to noise is also to be assessed.

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