

# Thin wire modelisation in the TLM method

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#### Abstract

The goal of the PhD is to adapt recent findings in a rival numerical scheme to the TLM (Transmission-Line Matrix) method. The principle is to model the electromagnetic wave propagated by an antenna without refining the mesh around the antenna wire. The utmost important criterion for the simulation to produce consistent results is to ensure the continuity of the discretized current throughout the mesh, and without which the simulation goes absolutely wrong. To further improve these results, an explicit numerical scheme proved more efficient than an implicit one, the sampling of the electric field surrounding the antenna needs to be taken away from the wire to avoid the singularity on its axis, and the computation of the inductance can be empirically modified to match the theoretical spectrum expected from the antenna.

#### 1 Introduction

In a recent article [1] from C. Guiffaut, A. Reineix and B. Pequeux, the problem of the thin wire in the FDTD mesh as been solved thanks to current continuity considerations. However, the paper doesn't give a theoretical explanation about their results, which as been made in [2] by Jean-Pierre Bérenger. The trickiest part to adapt these findings in the TLM methods comes from the fact that electromagnetic wave propagation in the TLM method are modeled through Transmission Lines. This is addressed in section 2. Further improvement can be made, in particular using an explicit numerical scheme (section 3), sampling the electric field further from the wire (section 4.1), and empirically adjusting the computation of the inductance (section 4.2).

#### **2** Current continuity

As shown in Figure 1a, the Symmetrical Condensed Node used in the TLM method models the propagation of electromagnetic waves using Transmission Lines. Therefore, the discretization of the Maxwell-Ampère equation (eq. 1) needs to take into account that the current moving from one cell to the other can only travel either along a straigh line in the case the wire crosses a TLM cell on two parallel faces, or at  $45^{\circ}$  when the wire enters by a face and exits by a perpendicular one (Figure 1b).

$$\nabla \wedge \vec{H} = \mu_0 \left( \vec{J} + \frac{\partial \vec{E}}{\partial t} \right) \tag{1}$$

This observation alone produces very consistent results, while when not fullfiled, the simulation gives completely errated responses from the antenna in the time domain as well as in the frequency domain.

#### 3 Implicit versus explicit numerical scheme

#### 3.1 Implicit numerical scheme

The SCN node used in the TLM method provides the values of both the Electric and Magnetic field at integer times, and at the center of the node. Therefore, the first idea is to synchronize the wire and the TLM grid, which results on an implicit formulation: the wire acts on the TLM cells at time n, and conversely the electric field acts on the wire at the same time which produces a matrix system to solve (eq. 2 and 3).

$$\begin{cases} {}_{k}I_{p} = \frac{2}{2\left(Z_{m} + Z_{\text{stub}}\right) + r\Delta\xi} \cdot \left(\frac{1}{2} \left[ {}_{k}E_{\vec{\xi}} + \tilde{v}^{\text{inc}} \right] + {}_{k}V^{i}_{\text{w}, \ \tilde{\xi}, \ 1} - {}_{k}V^{i}_{\text{w}, \ \tilde{\xi}, \ r} + 2{}_{k}V^{i}_{\text{w}, \ \tilde{\xi}, \ s} \right) \tag{2}$$

$$\int_{k} E_{\vec{\xi}} = \Delta \xi \sum_{\text{cells } q} \left( \frac{\alpha}{\Delta x} \left( E_{xq} - {}_{k}I_{q} \cdot \alpha \right) + \frac{\beta}{\Delta y} \left( E_{yq} - {}_{k}I_{q} \cdot \beta \right) + \frac{\gamma}{\Delta z} \left( E_{zq} - {}_{k}I_{q} \cdot \gamma \right) \right)$$
(3)



Figure 1: SCN node, enforcement of the electric field, and comparison between implicit and explicit schemes.

However, the result is a bit far from the theoretical spectrum that can obtained with the Method Of Moments, which can be improve with an explicit scheme.

### 3.2 Explicit numerical scheme

The SCN node, however, allows to rebuild the electric field at each interface along two coordinates at a time, at time half-integer  $n + \frac{1}{2}$ . This observation can be used to set up an explicit scheme, the wire running at half-integer times and the TLM cells at integer times. The comparison between both implicit and explicit spectrums are provided in Figure 1c.

### 4 Electric field sampling and inductance computation

### 4.1 Electric field sampling

Since the media acts on the wire, the sampling of the electric field should be done spontaneously in the cells crossed by the wire. However, considering 9 cells around the wire in a plane perpendicular to the direction of a field component ( $E_x$ ,  $E_y$  or  $E_z$ ), and by sampling only the 4 cells at the corners, the results happen to be much better.

#### 4.2 Inductance computation

The inductance computation as proposed in [1] can be empirically divided by 2.25 to further superimpose the spectrum to the one retrieved by the Method of Moments. Despite the fact this correction is not justified by the theory, the results are the best obtained so far.

## 5 Conclusion

The main discovery in this work is the fact that the TLM cells need to be enforced by the current density either along an axis or at  $45^{\circ}$ . This ensures the charge conservation law is fullfiled.

## 6 Acknowlegments

The work done in this PhD has been made possible by Prof. Reineix and Prof. Guiffaut who solved the thin wire problem in the FDTD method, and by Prof. Berenger who found the theoretical explanation to the solution. Also, a particular acknolegment must be made to the Paraview software development team from the Kitware company, for their tool allowed investigations by no comparable means more efficient than print tracing to adapt the findings in the FDTD scheme to the TLM one.

## 7 References

- 1. Guiffaut C., Reineix A., and Pecqueux B., New oblique thin wire formalism in the FDTD method with multiwire junction, IEEE Transactions on Antennas and Propagation, March 2012.
- 2. Bérenger J.-P., Origin of Parasitic Solutions With Holland and Simpson Thin Wires in the FDTD Grid, IEEE Transactions on Electromagnetic Compatibility, 2018.