Cognitive Radio – A Necessity for Spectrum Pooling

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Paris, March 28, 2006

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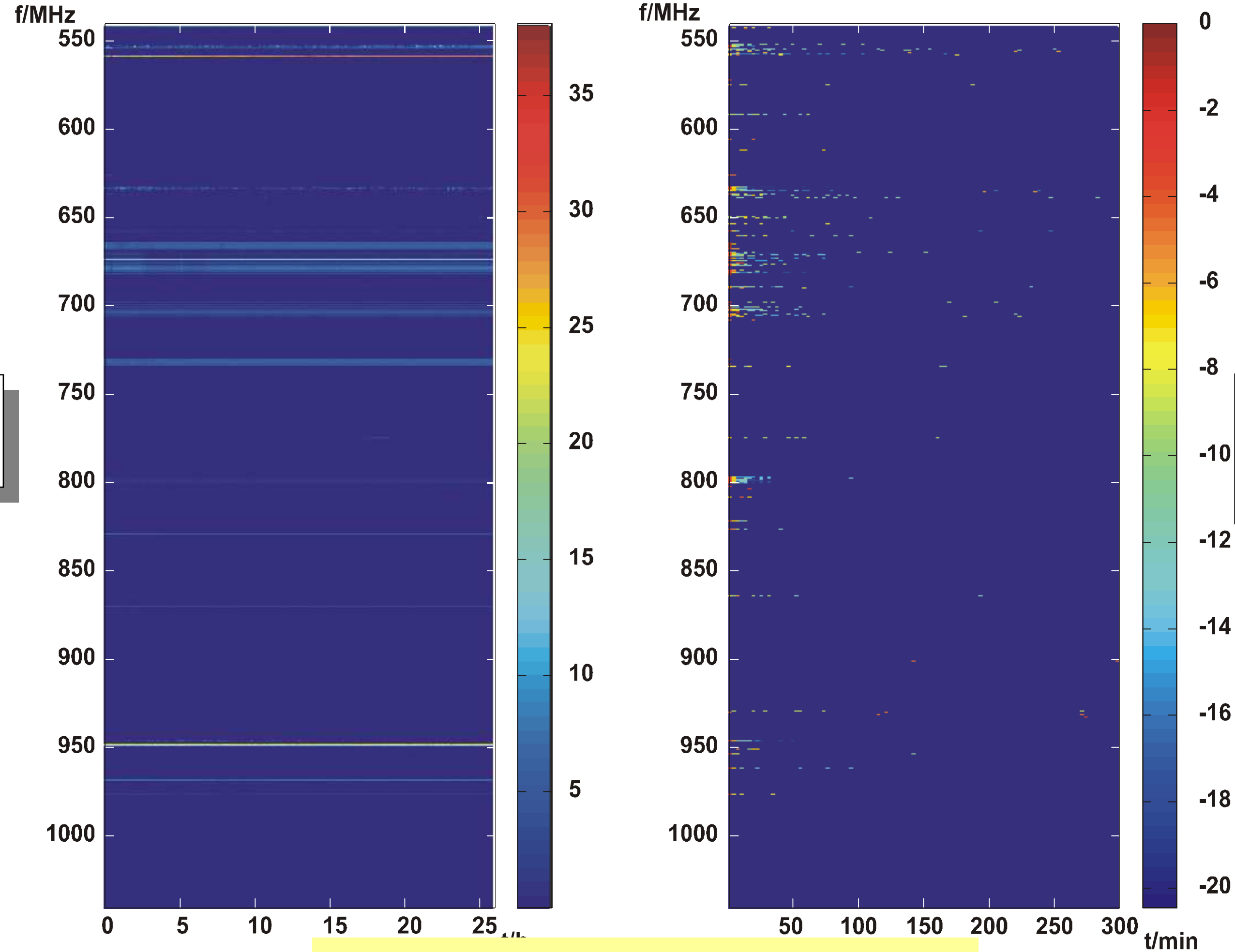
Spectrum Utilization Measurements
(550-1000MHz)

electric field strength dBµV/m



density of the time between arrivals dBs^{-1}

Lichtenau (Germany), September 2001

Synopsis

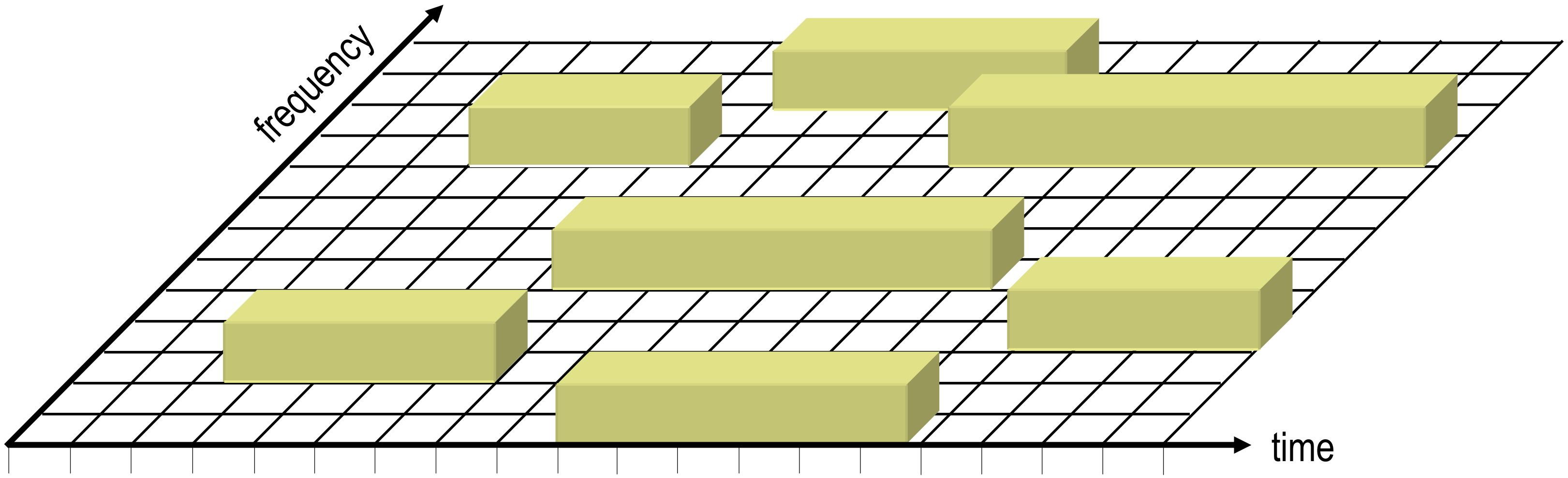
- **Spectrum Pooling**
- **HIPERLAN/2 System Overview**
- **The Licensed User (LU) System**
- **Rental User (RU) System: Physical Layer Issues**
- **Rental User (RU) System: Detection and Signaling**
- **Cognitive Radio**

Spectrum Pooling

Vision:

Usage of free capacities in licensed frequency bands:

***Licensed Users* (LUs) lease out spectrum to *Rental Users* (RUs)**

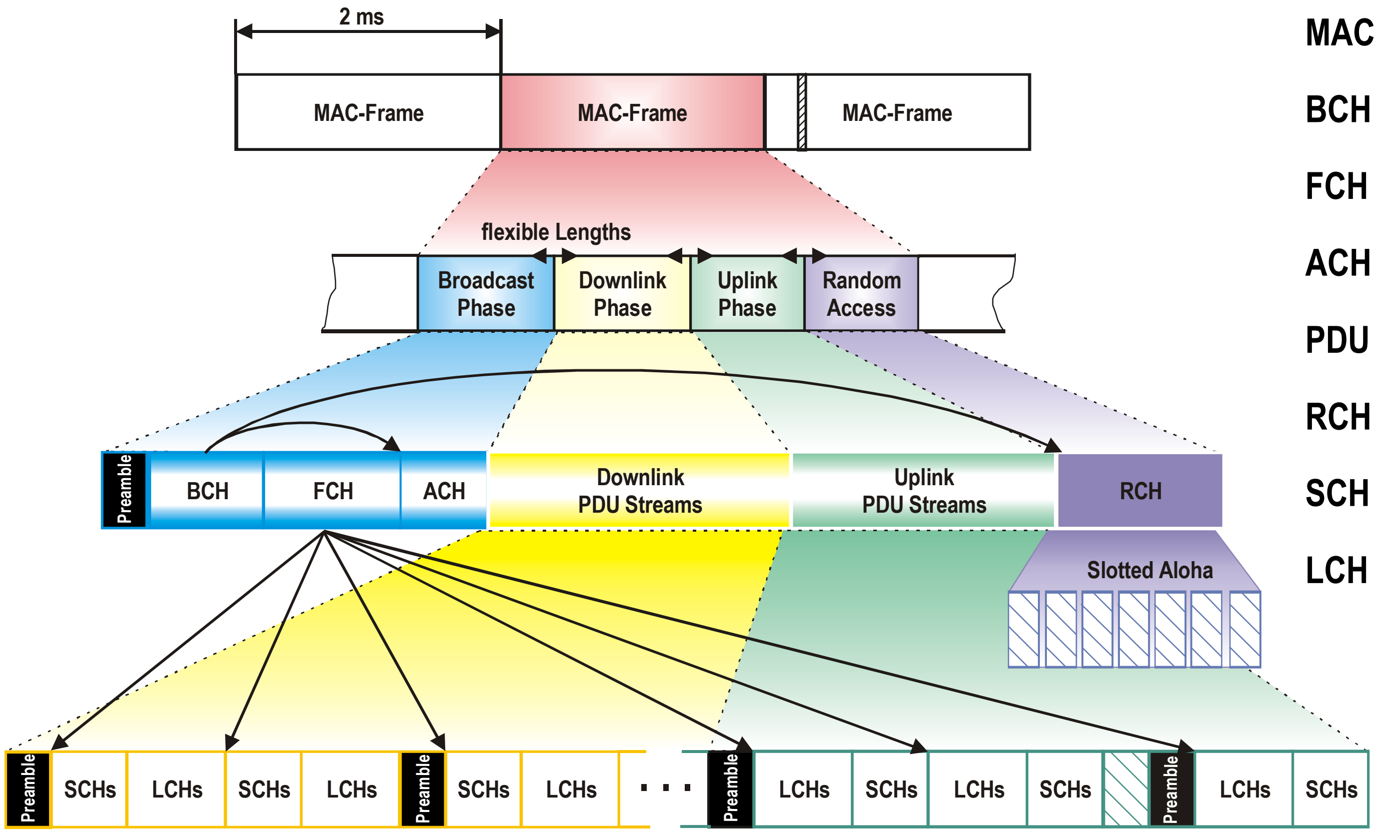


FDMA / TDMA LU system and HIPERLAN/2 RU system:

Two different radio systems → Coexistence in the same frequency region ?!

HIPERLAN/2 System Overview

- **European Wireless Local Area Network Standard**



MAC *Medium Access Control*

BCH *Broadcast CHannel*

FCH *Frame CHannel*

ACH *Access feedback CHannel*

PDU *Protocol Data Unit*

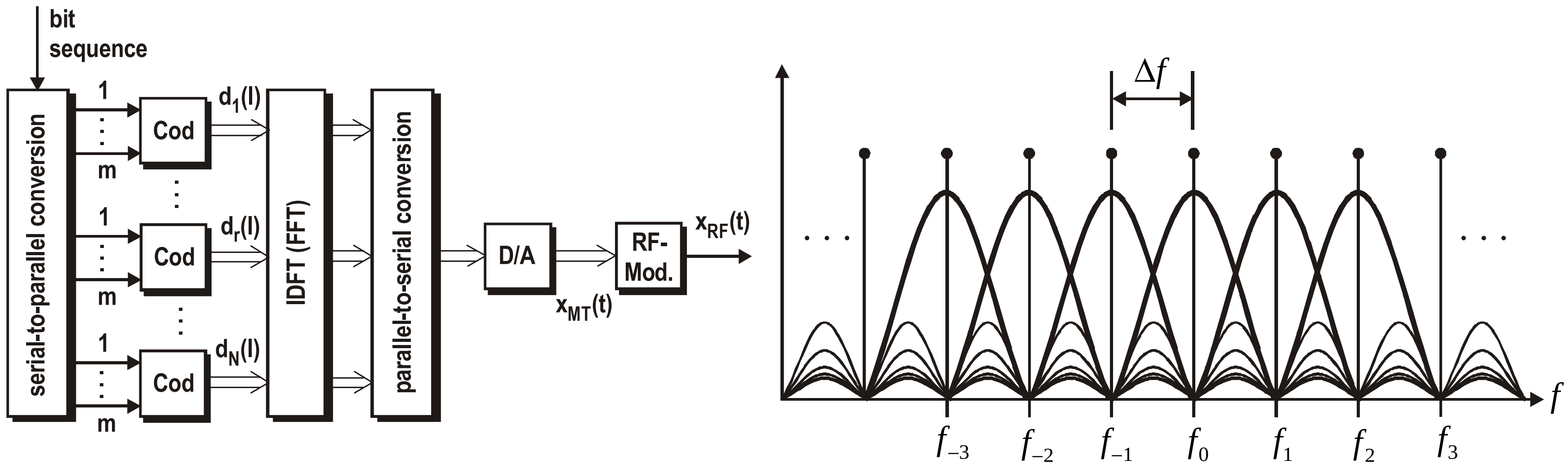
RCH *Random CHannel*

SCH *Short transport CHannel*

LCH *Long transport CHannel*

HIPERLAN/2 System Overview

- **Physical Layer: OFDM**
  Transmitter structure and spectrum

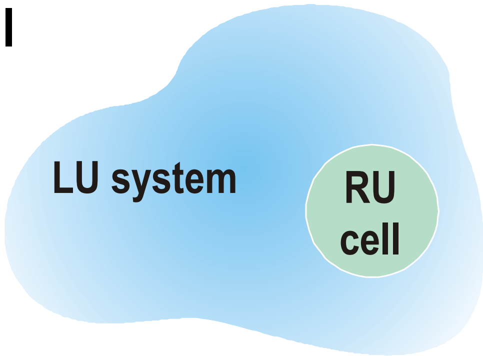


HIPERLAN/2 System Overview

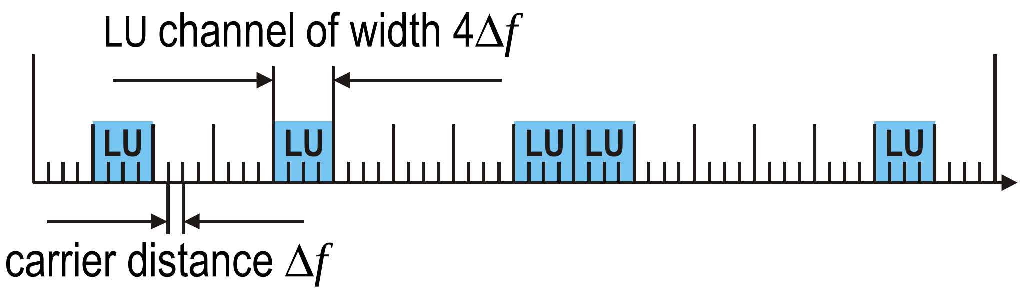
Total Number of Subcarriers	**64**
Number of Data Subcarriers K_D	**48**
Number of Pilots K_P	**4**
Total Number of OFDM Subcarriers Used K	**52 (= K_D + K_P)**
FFT Bandwidth	**20 MHz**
OFDM Subcarriers‘ Distance Δf	**312,5 kHz (= 20 MHz / 64)**
Time Distance Between two FFT Input Samples T_A	**50 ns (= 1/20 MHz)**
Usable Part of an OFDM Symbol T_U	**64 T_A = 3.2 µs (= 1/ Δf)**
Guard Interval T_G	**16 T_A = 0.8 µs**
OFDM Symbol	**4 µs (= T_U + T_G)**

The Licensed User System

1. **Embedding a RU cell into a LU cell (Hot Spot Scenario)**



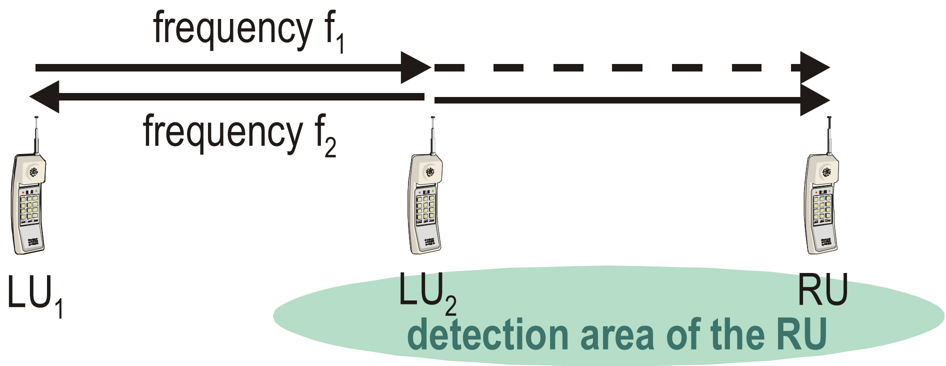
2. **Channel pattern and Occupancy Vector (OV)**



OV = (0 1 0 0 1 0 0 0 1 1 0 0 0 0 1 0)

The Licensed User System

3. **No LU signaling channel → LU detection necessary for the RU system**
   - **hidden / exposed station problem**



➔ **The detection has to cover the whole sphere of influence of the RU cell**

➔ **Access delay for the LUs**

➡ **Waiting period**

4. **No Carrier Sense Medium Access (CSMA) in the LU system.**

RU Physical Layer Issues

- **LU detection and signaling**
  - **Optimum detection?**
  - **Quality of detection necessary for coexistence?**
  - **OV transmission calls for a new protocol**

- **RU system synchronization**
  - Necessity of a new preambel concept
  - Optimum positioning of pilots?

- **Interference reducing measures**
  - Disturbances to the LU system caused by the ***sin(x)/x***-shaped OFDM spectrum
  - Disturbances to the RU system by FFT leakage
    caused by the non orthogonality of the LU signals

Detection and Signaling

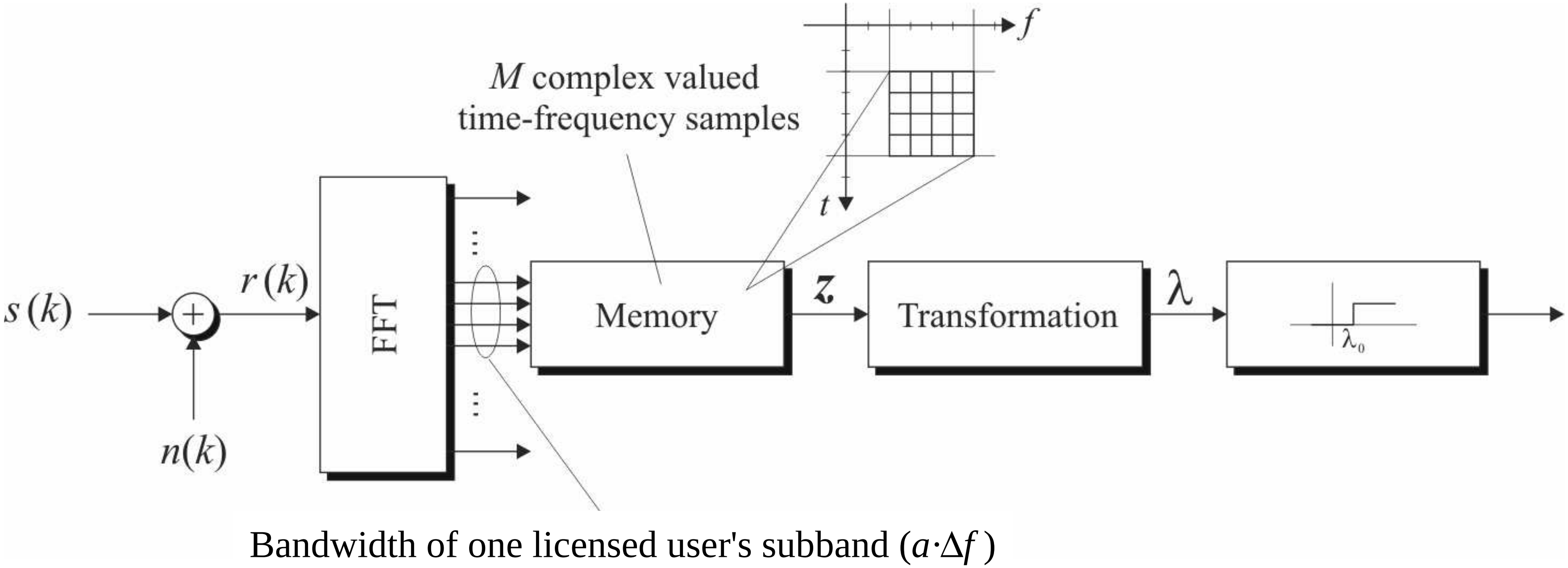
The engaged / idle decision has to be done by the RU system for each LU channel within the pool

➡ The frequency resolution is realized by the anyhow existing FFT:

1. **Sampling of the signal $s(k)$ band limited to the pool width**
2. **FFT for 64 samples at a time. The process is repeated n times.**
3. **The spectrum values belonging to one LU channel are integrated into a vector z .**
4. **Decision based on z and on an optimality criterion.**

Signal and Detector Model

Structure of the detector in the rental user's station



$$\mathbf{z} = (\underbrace{x_1, x_2, \ldots, x_M}_{M \text{ real parts}}, \underbrace{y_1, y_2, \ldots, y_M}_{M \text{ imaginary parts}})^T$$

Joint Conditional PDFs

NLOS assumption and central limit theorem yield:

$$f_{\mathbf{Z}|\mathrm{LU}}(\mathbf{z}|\mathrm{LU}) = \left[(2\pi)^{2M}\det\left(\mathbf{C}_{\mathbf{SS}}+\sigma_N^2\mathbf{I}\right)\right]^{-\frac{1}{2}} \cdot \exp\left(-\frac{1}{2}\mathbf{z}^T\left(\mathbf{C}_{\mathbf{SS}}+\sigma_N^2\mathbf{I}\right)^{-1}\mathbf{z}\right)$$

Only noise in case of licensed user's absence:

$$f_{\mathbf{Z}|\overline{\mathrm{LU}}}(\mathbf{z}|\overline{\mathrm{LU}}) = \frac{1}{(2\pi\sigma_N^2)^M}\exp\left(-\frac{\mathbf{z}^T\mathbf{z}}{2\sigma_N^2}\right)$$

Likelihood ratio test according to the NEYMAN-PEARSON-criterion:

$$\frac{f_{\mathbf{Z}|\mathrm{LU}}(\mathbf{z}|\mathrm{LU})}{f_{\mathbf{Z}|\overline{\mathrm{LU}}}(\mathbf{z}|\overline{\mathrm{LU}})} = \Lambda(\mathbf{z}) \underset{\overline{\mathrm{LU}}}{\overset{\mathrm{LU}}{\gtrless}} \lambda_0$$

Detector for Uncorrelated Samples

Under the condition $\mathbf{C}_{\mathbf{SS}} = \sigma_S^2 \cdot \mathbf{I}$, the PDF $f_{\mathbf{Z}|\mathrm{LU}}(\mathbf{z}|\mathrm{LU})$ simplifies to:

$$f_{\mathbf{Z}|\mathrm{LU}}(\mathbf{z}|\mathrm{LU}) = \left(2\pi(\sigma_S^2 + \sigma_N^2)\right)^{-M} \exp\left(-\frac{\mathbf{z}^T\mathbf{z}}{2(\sigma_S^2 + \sigma_N^2)}\right)$$

With these conditional PDFs the likelihood ratio results in:

$$\Lambda(\mathbf{z}) = \mathbf{z}^T\mathbf{z} = \sum_{m=1}^{M} x_m^2 + \sum_{m=1}^{M} y_m^2 \underset{\mathrm{LU}}{\overset{\mathrm{LU}}{\gtrless}} \lambda_0$$

Considering the likelihood ratio as a transformation of random variables:

$$\begin{aligned} \Lambda(\mathbf{Z}) &= \sum_{m=1}^{M} X_m^2 + \sum_{m=1}^{M} Y_m^2 \\ &= \sum_{m=1}^{M} (S_{X_m} + N_{X_m})^2 + \sum_{m=1}^{M} (S_{Y_m} + N_{Y_m})^2 \end{aligned}$$

Worst Case Consideration

Maximum deviation from the model, if real/imaginary parts are fully correlated:

$$\mathbf{C_{SS}} = \begin{pmatrix} \mathbf{C_{XX}} & \mathbf{0} \\ \mathbf{0} & \mathbf{C_{YY}} \end{pmatrix} = \begin{pmatrix} \mathbf{C_{XX}} & \mathbf{0} \\ \mathbf{0} & \mathbf{C_{XX}} \end{pmatrix} \text{ mit } \mathbf{C_{XX}} = \sigma_S^2 \begin{pmatrix} 1 & \cdots & 1 \\ \vdots & \ddots & \vdots \\ 1 & \cdots & 1 \end{pmatrix}$$

Hence, all S_{X_m} are identical and can be represented by S_X:

$$\Lambda(\mathbf{Z}) = \sum_{m=1}^{M} \Big(\underbrace{S_X + N_{X_m}}_{X_m} \Big)^2 + \sum_{m=1}^{M} \Big(\underbrace{S_Y + N_{Y_m}}_{Y_m} \Big)^2$$

$\Lambda(\mathbf{Z})$ can be interpreted as a concatenated random variable:

$$f_{\Lambda|\mathrm{LU}}(\lambda|\mathrm{LU}) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_{S_X}(s_X) \cdot f_{S_Y}(s_Y) \cdot f_{\Lambda|S_X,S_Y}(\lambda|s_X, s_Y)\, ds_X\, ds_Y$$

Detection Probability

$f_{\Lambda|\mathrm{LU}}(\lambda|\mathrm{LU})$ is the marginal PDF with respect to a randomized χ^2- distribution:

$$f_{\Lambda|\mathrm{LU}}(\lambda|\mathrm{LU}) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{1}{4\pi\sigma_S^2\sigma_N^2}\left(\frac{\lambda}{M(s_X^2+s_Y^2)}\right)^{\frac{M-1}{2}} \cdot \exp\left(-\frac{\lambda+M(s_X^2+s_Y^2)}{2\sigma_N^2}\right)$$

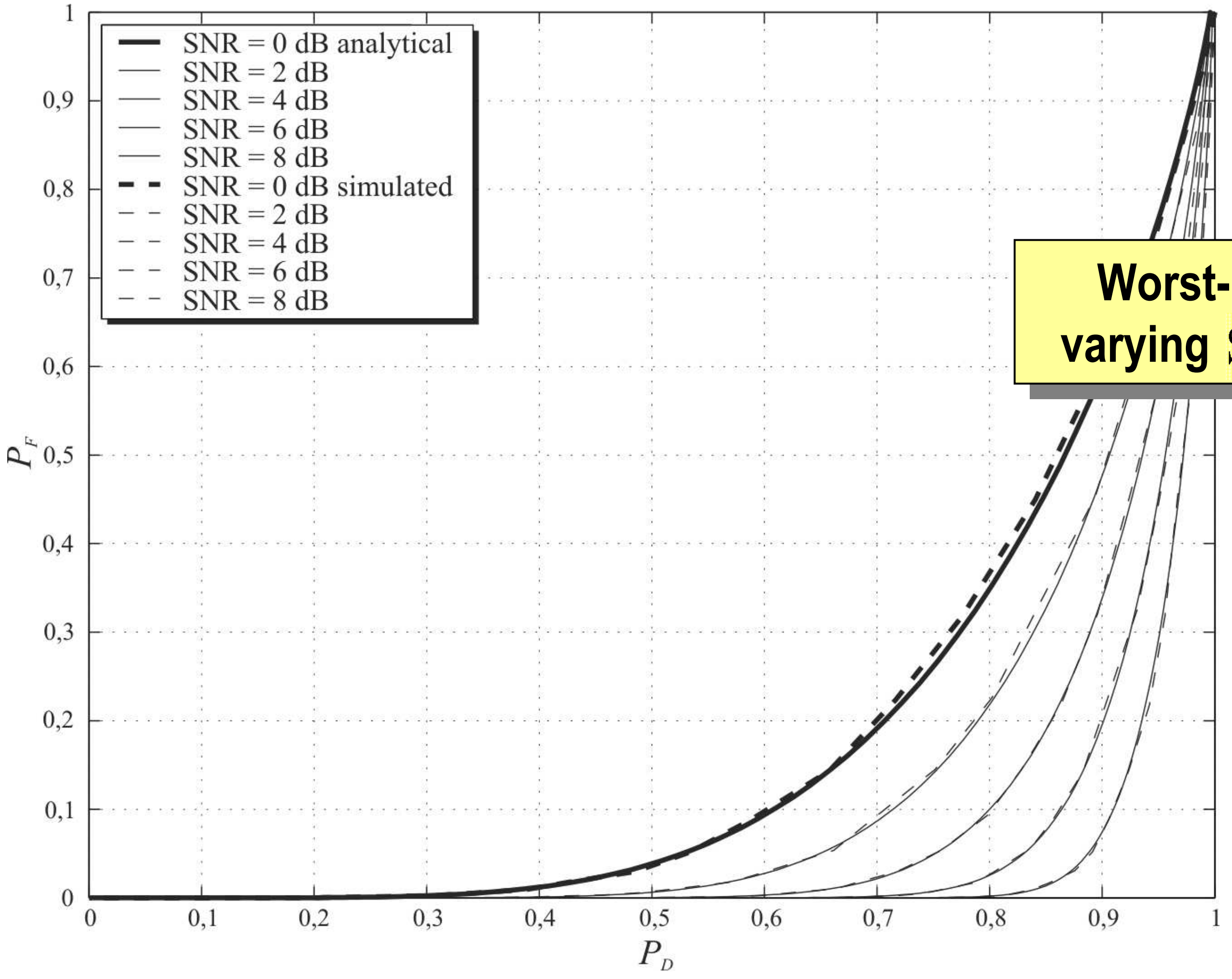
$$\cdot I_{M-1}\left(\frac{\sqrt{M(s_X^2+s_Y^2)\lambda}}{\sigma_N^2}\right)\cdot\exp\left(-\frac{s_X^2+s_Y^2}{2\sigma_S^2}\right) ds_X\, ds_Y$$

The detection probability $P_D = \int_{\lambda_0}^{\infty} f_{\Lambda|\mathrm{LU}}(\lambda|\mathrm{LU})\, d\lambda$ can be calculated as:

$$P_D = \left(\frac{\sigma_N^2+M\sigma_S^2}{M\sigma_S^2}\right)^{M-1}\cdot\left[\exp\left(-\frac{\lambda_0}{2\sigma_N^2+2M\sigma_S^2}\right)-\frac{1}{2\sigma_N^2+2M\sigma_S^2}\exp\left(-\frac{\lambda_0}{2\sigma_N^2}\right)\right.$$

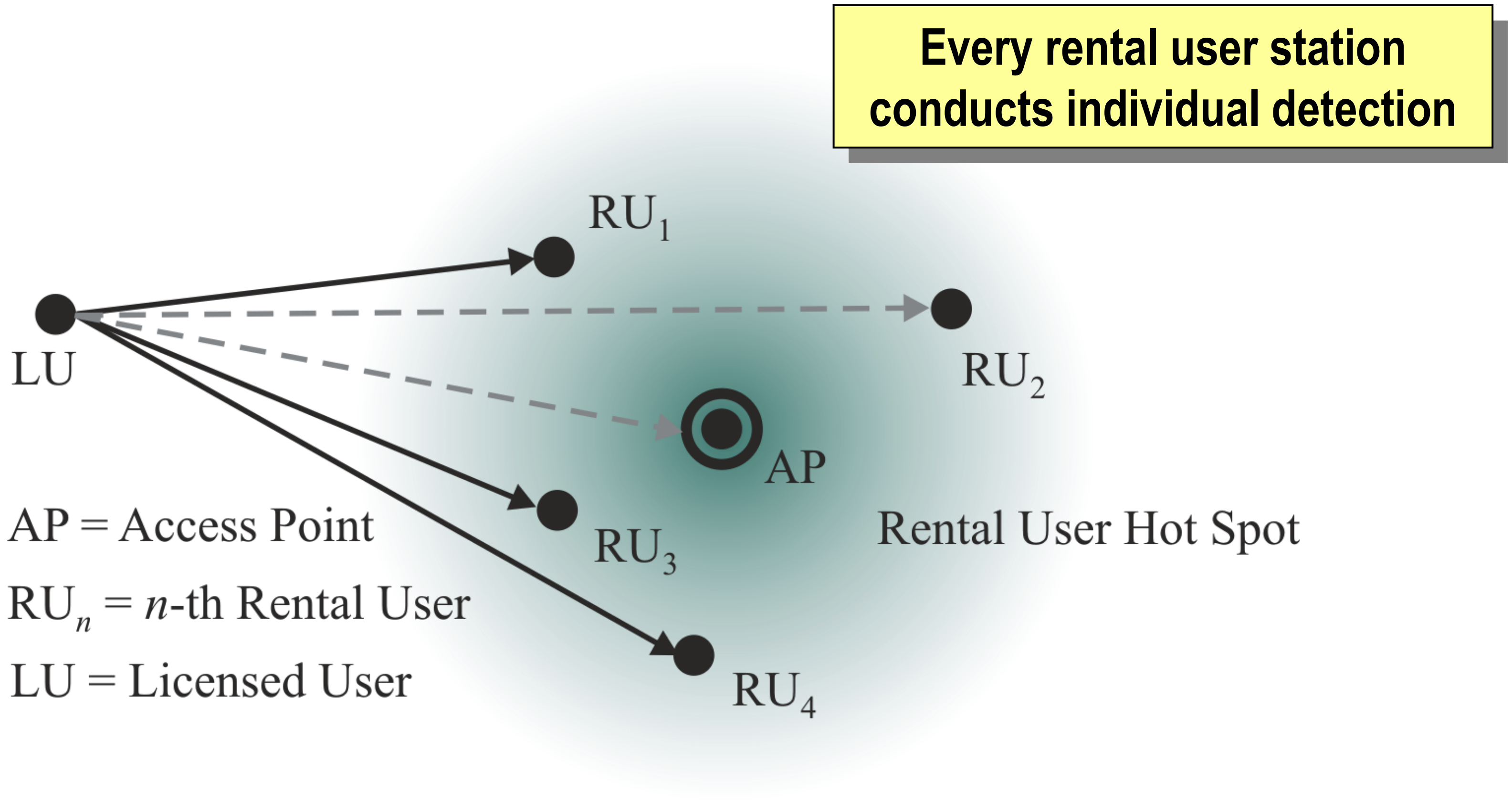
$$\left.\cdot\sum_{m=0}^{M-2}\left(\frac{M\sigma_S^2}{2\sigma_N^2(\sigma_N^2+M\sigma_S^2)}\right)^m\sum_{l=0}^{m}\frac{(2\sigma_N^2)^{l+1}\lambda_0^{m-l}}{(m-l)!}\right]$$

Receiver Operating Characteristic (ROC)



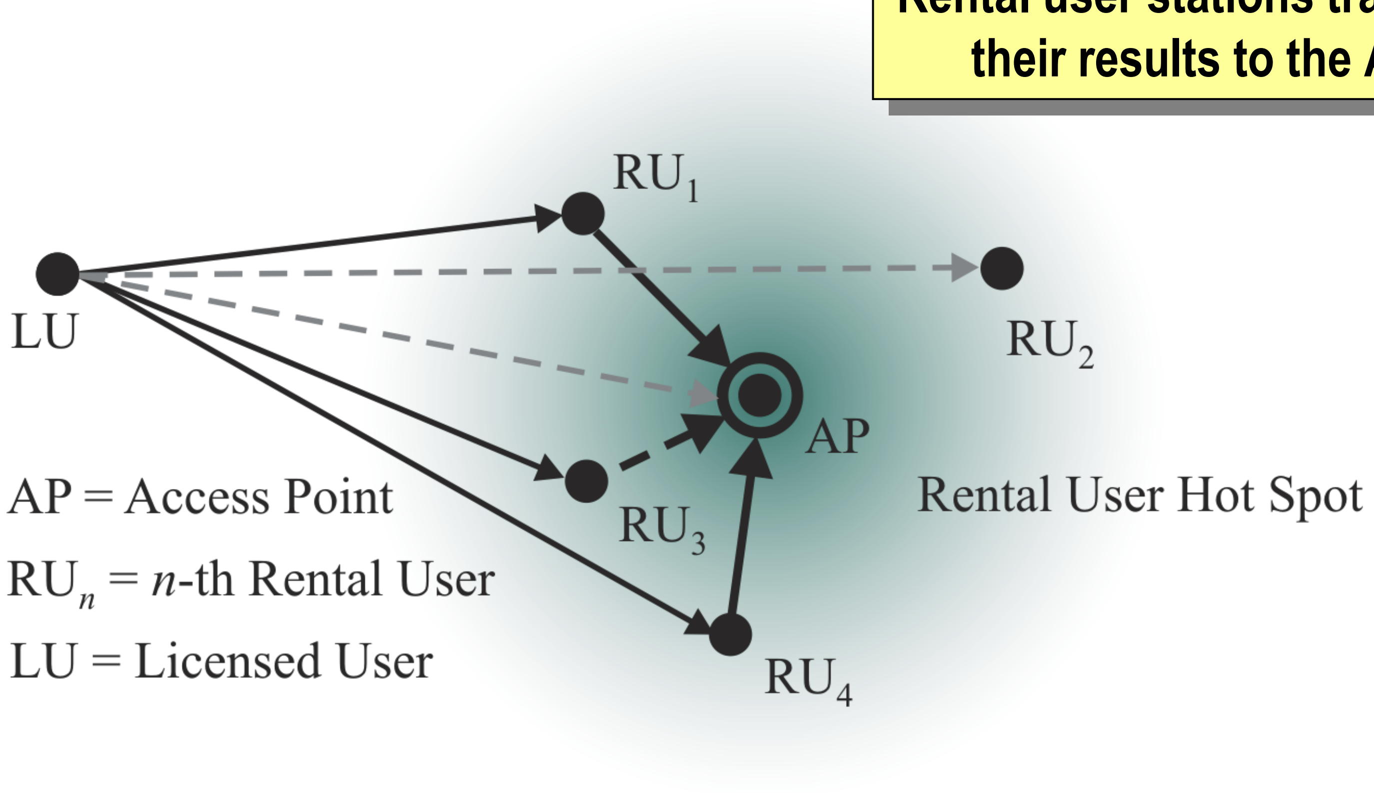
Worst-case ROC for varying $\mathrm{SNR} = \sigma_S^2/\sigma_N^2$

Distributed Detection: Single Detection



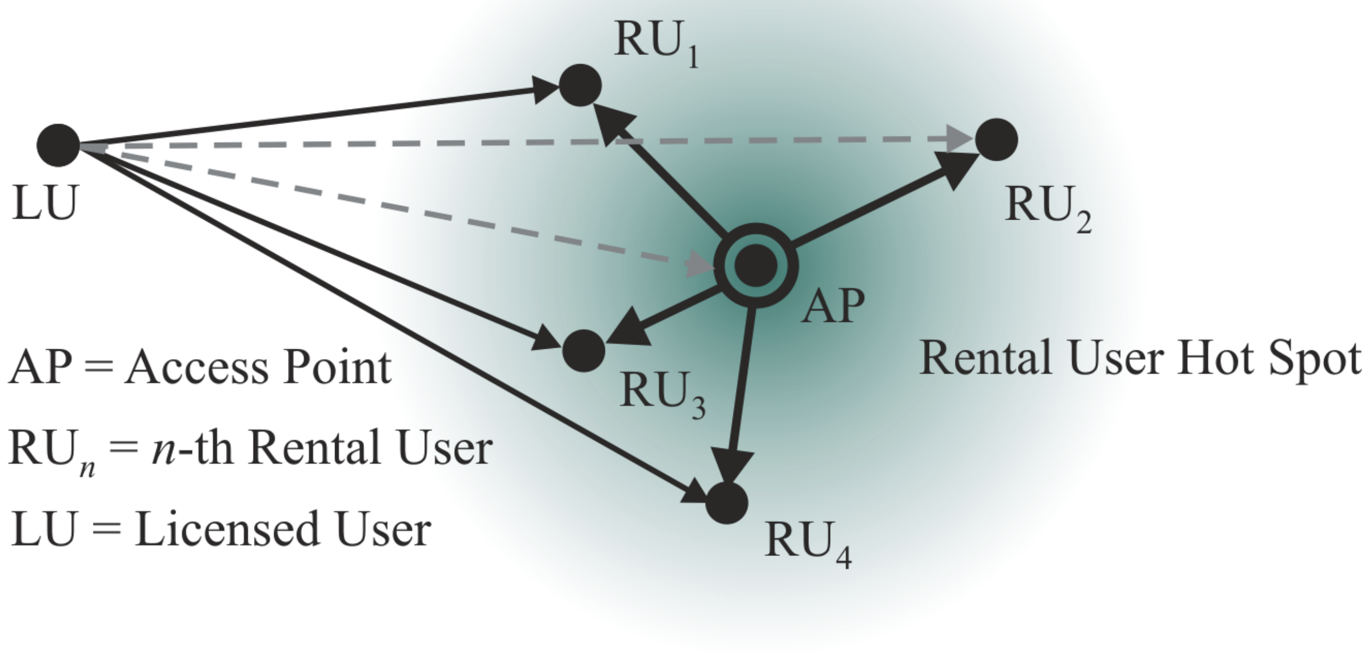
Distributed Detection: Collection

Rental user stations transmit their results to the AP



Distributed Detection: Broadcast

After an OR-combination, the AP broadcasts the new allocation vector



Gain by Distributed Detection

Overall Probability of detection if *at least one* of the N stations detects the LU access:

$$\begin{aligned}\tilde{P}_D(N) &= 1-(1-P_D)^N \quad \text{and}\\ \tilde{P}_F(N) &= 1-(1-P_F)^N\end{aligned}$$

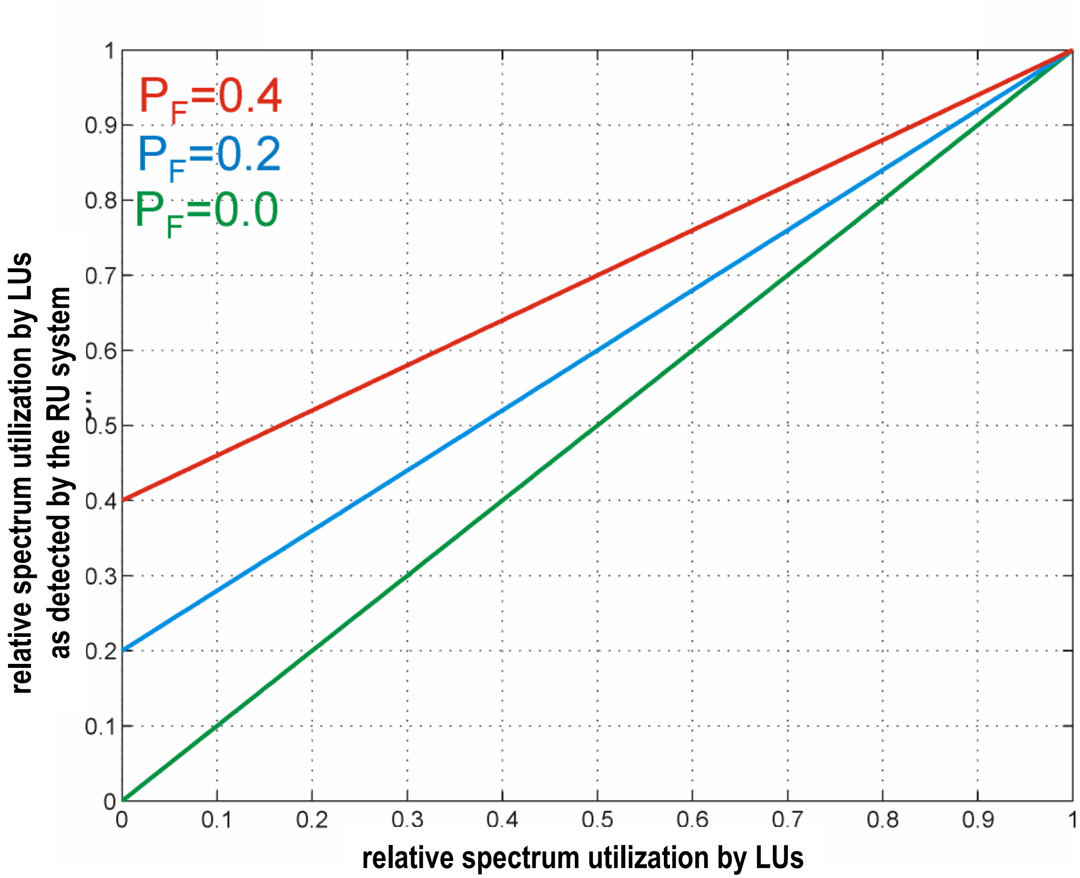
Behavior of the overall false alarm probability $\tilde{P}_F$ if $\tilde{P}_D$ is predetermined:

N	P_D	P_F	$\tilde{P}_F$
1	0,999	0,982	0,982
2	0,968	0,662	0,886
3	0,900	0,294	0,648
4	0,822	0,100	0,344
5	0,749	0,034	0,159
10	0,499	0,001	0,010
20	0,292	≈ 0	≈ 0

$\tilde{P}_D = 0{,}999$
$M = 16$
$\text{SNR} = 3\ \text{dB}$

Spectrum Efficiency

- **Impact of the false alarm probability P_F on the RU system efficiency**

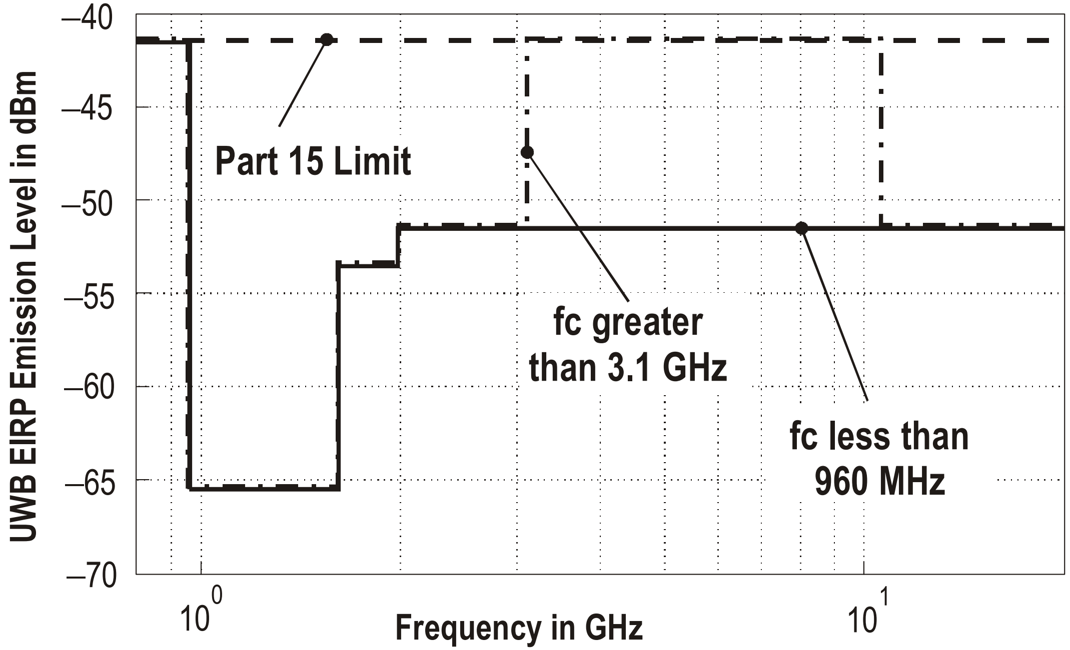


Regulation

- Today "spectrum" is regulated by governmental agencies, e.g. the American Federal Communications Commission (FCC) or the German Regulierungsbehörde für Telekommunikation und Post (RegTP)
- "Spectrum" is assigned to users or licensed to them on a long term basis normally for huge regions like whole countries
- Doing so, resources are wasted
- Vision: Resources are assigned where and as long as they are needed, spectrum access is organized by the network (i.e. by the end users)

Self Regulation

- **Wireless LANs (IEEE 802.11x)**
  ISM band: 2400 – 2483.5 MHz
  WLAN band: 5150 – 5350 MHz and 5470 – 5725 MHz
- **Ultra Wide Band**



Advanced Spectrum Management

- **Spectrum reallocation: The reallocation of bandwidth from government or other long-standing users to new services such as mobile communications, broadband internet access, and video distribution.**
- **Spectrum leases: The relaxation of the technical and commercial limitations on existing licensees to use their spectrum for new or hybrid (for example, satellite and terrestrial) services and granting most mobile radio licensees the right to lease their spectrum to third parties.**
- **Spectrum sharing: The allocation of an unprecedented amount of spectrum that could be used for unlicensed or shared services.**

According to

G. Staple, K. Werbach: *The End of Spectrum Scarcity.* IEEE Spectrum, March 2004, pp. 41-44

Cognitive Radio

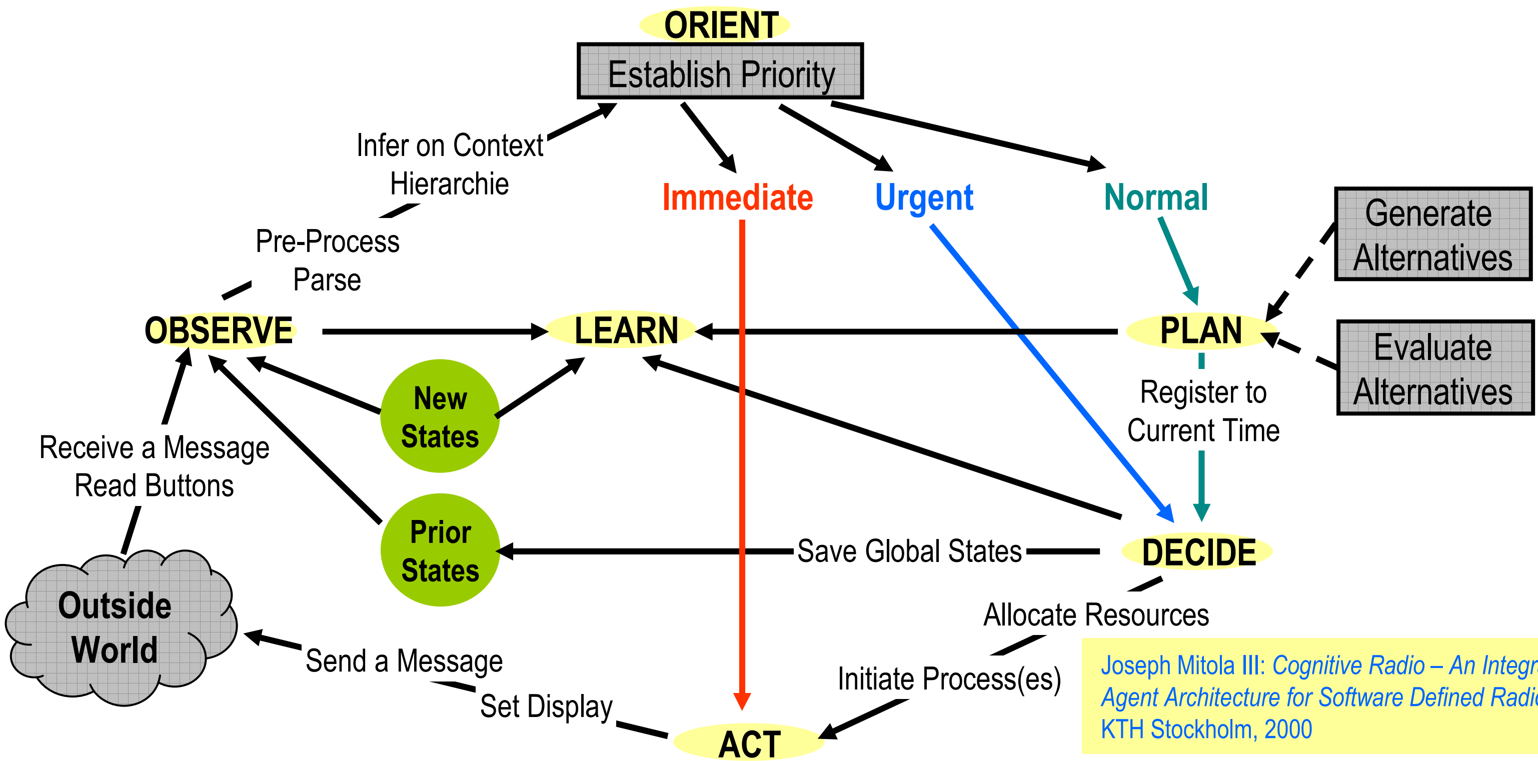
- **A Cognitive Radio (CR) is an SDR that additionally senses its environment, tracks changes and reacts upon its findings.**

- **A CR is an autonomous unit in a communications environment. In order to use the spectral resource most efficiently, it has to**
  - **be aware of its location**
  - **be interference sensitive**
  - **comply with some communications etiquette**
  - **be fair against other users**
  - **keep its owner informed**

Cognition Cycle

A necessary condition for highest flexibility in mobile communications is a general rethinking in spectrum allocation: Open access

In order to make open access feasible Cognitive Radios are necessary.



Joseph Mitola III: *Cognitive Radio – An Integrated Agent Architecture for Software Defined Radio.* KTH Stockholm, 2000

CR Properties

Mitola's cognition cycle is very general. The properties of cognitiv radios may be divided into two groups

user centric properties (support functions like finding an appropriate restaurant, recommendation of a travel route, supervision of apointments, . . .)

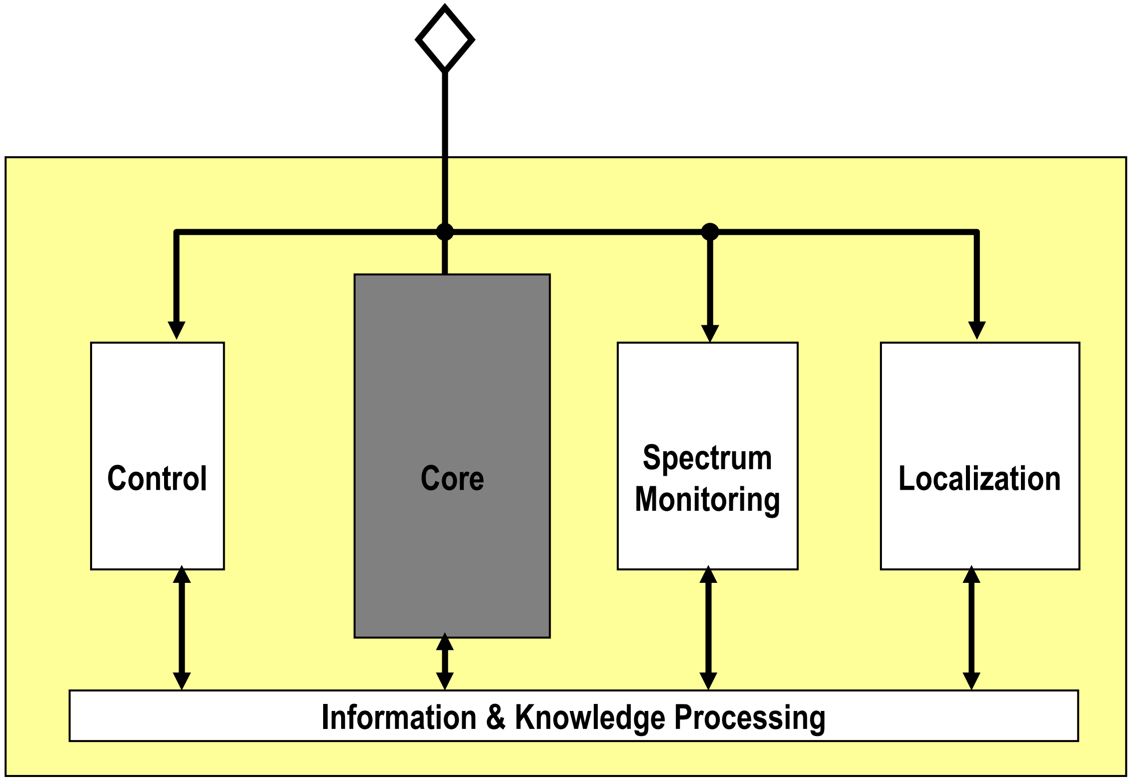
technology centric properties

- **spectrum monotoring**
- **localization**
- **awareness of processing capabilities (partitioning and scheduling of processes)**
- **information and knowledge processing**

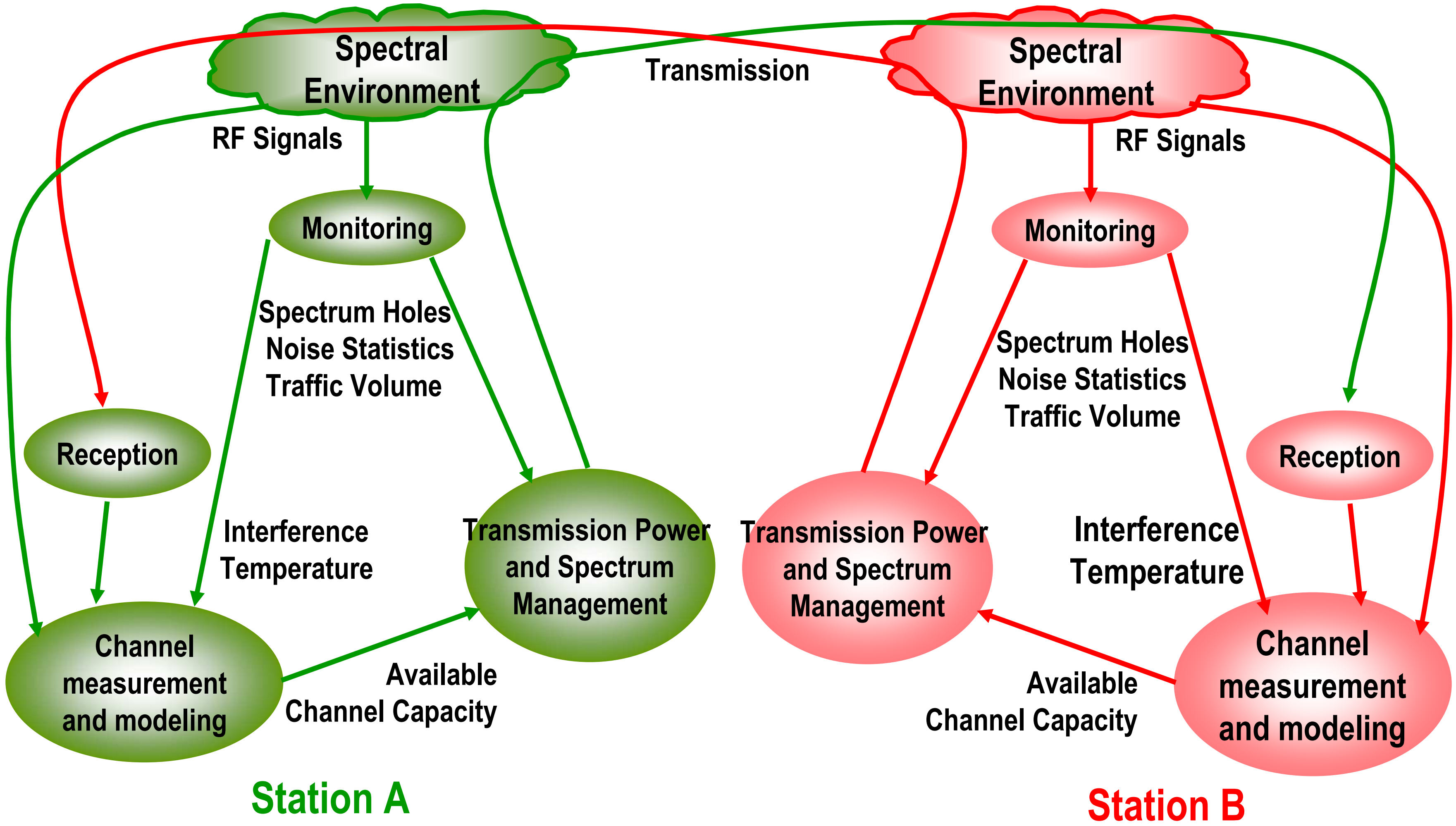
Technologies to be Implemented

- A CR carries location (e.g. GPS or Galileo) sensors.
- It has to monitor its spectral environment, for example by using a real time broadband FFT.
- In order to track its location or the spectral environment‘s development, it has to implement learning and reasoning algorithms.
- When complying with a communications etiquette it has to listen before talk as well as to prevent the disturbance of hidden stations.
- In order to be fair it has to compromise its own demands with the demands of other users, most probably making decisions in a competitive environment using the results of game theory.
- It has to contact its owner via a highly sophisticated man-machine-interface.

Technology Centric CR



TeC-CR


Spectral Environment
Transmission
Spectral Environment
RF Signals
RF Signals
Monitoring
Monitoring
Spectrum Holes
Noise Statistics
Traffic Volume
Spectrum Holes
Noise Statistics
Traffic Volume
Reception
Reception
Interference
Temperature
Transmission Power and Spectrum Management
Transmission Power and Spectrum Management
Interference
Temperature
Channel measurement and modeling
Available
Channel Capacity
Available
Channel Capacity
Channel measurement and modeling
Station A
Station B

Conclusions

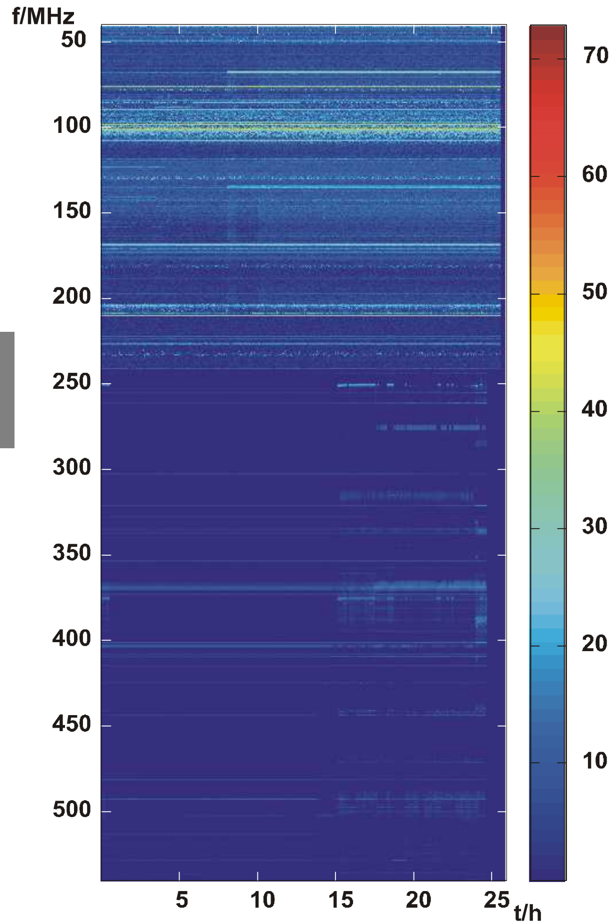
1. **Advanced spectrum management will be a hot topic in future research on wireless networks.**
2. **Unlicensed (ISM, WLAN) as well as secondary (UWB) spectrum usage are already under way.**
3. **First spectrum sharing strategies (e.g. spectrum pooling) are under investigation.**
4. **Advanced spectrum management calls for new developments in networking and terminal devices:**

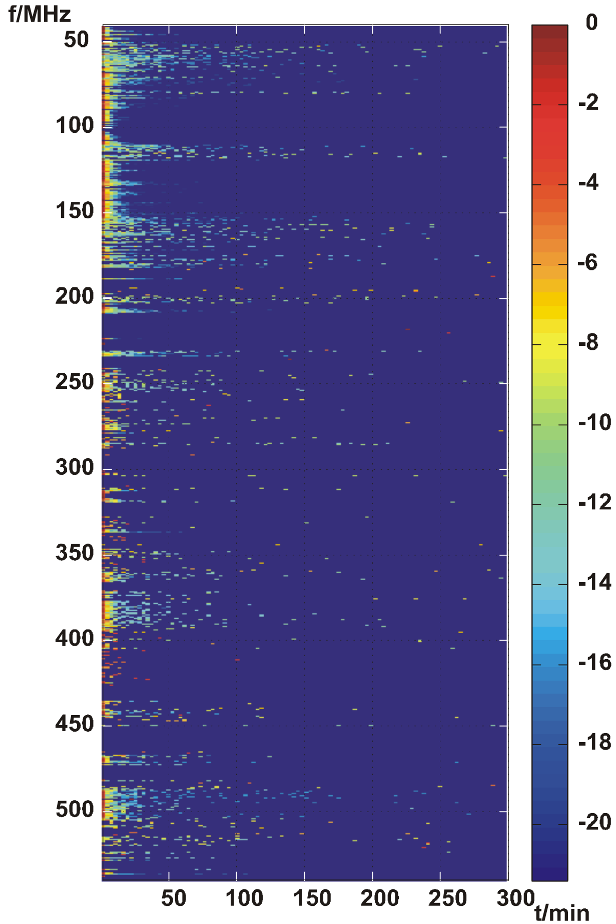
Intelligent Networks & Cognitive Radios

Questions ?

SPECTRUM UTILIZATION MEASUREMENTS (50-550 MHz)

electric field strength dBμV/m

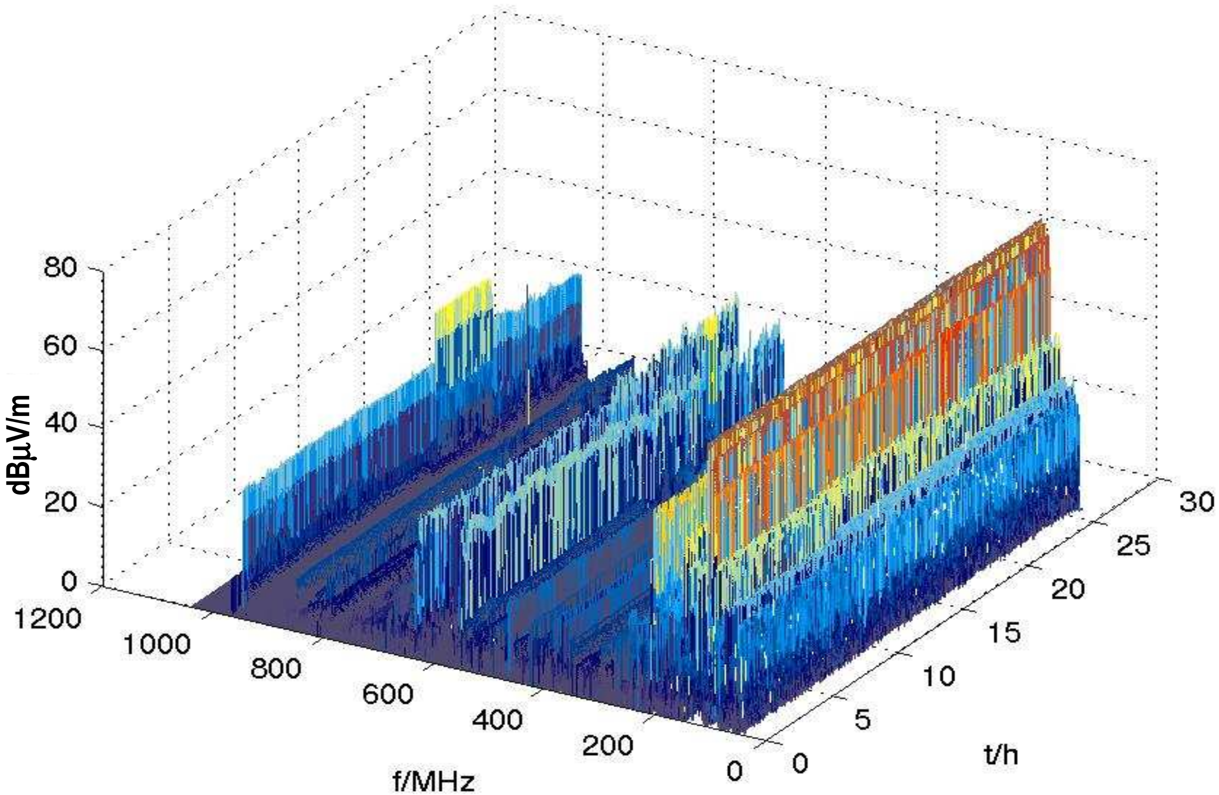




density of the time between arrivals dBs^{-1}

Lichtenau (Germany), September 2001

SPECTRUM UTILIZATION
(50 MHz-1GHz)



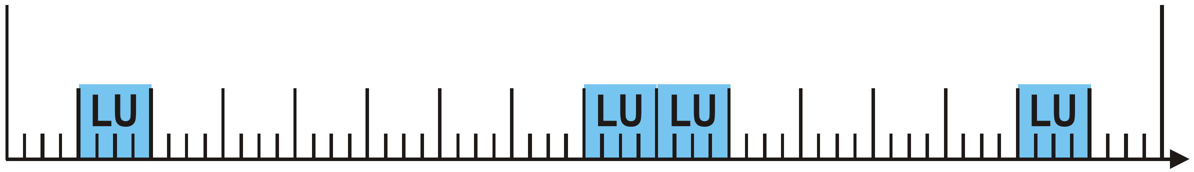
Lichtenau (Germany), September 2001

BOOSTING PROTOCOL (1)

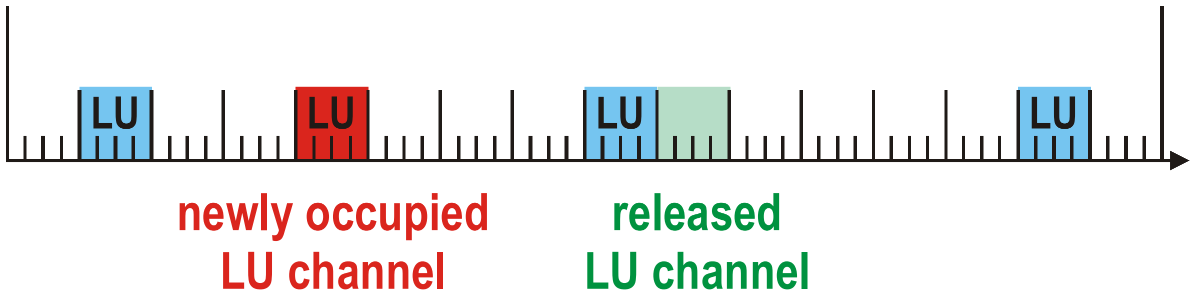
- **Signaling RUs + BooSs $\longrightarrow$ AP ?**

- **Boosting Protocol**

Presently valid OV:



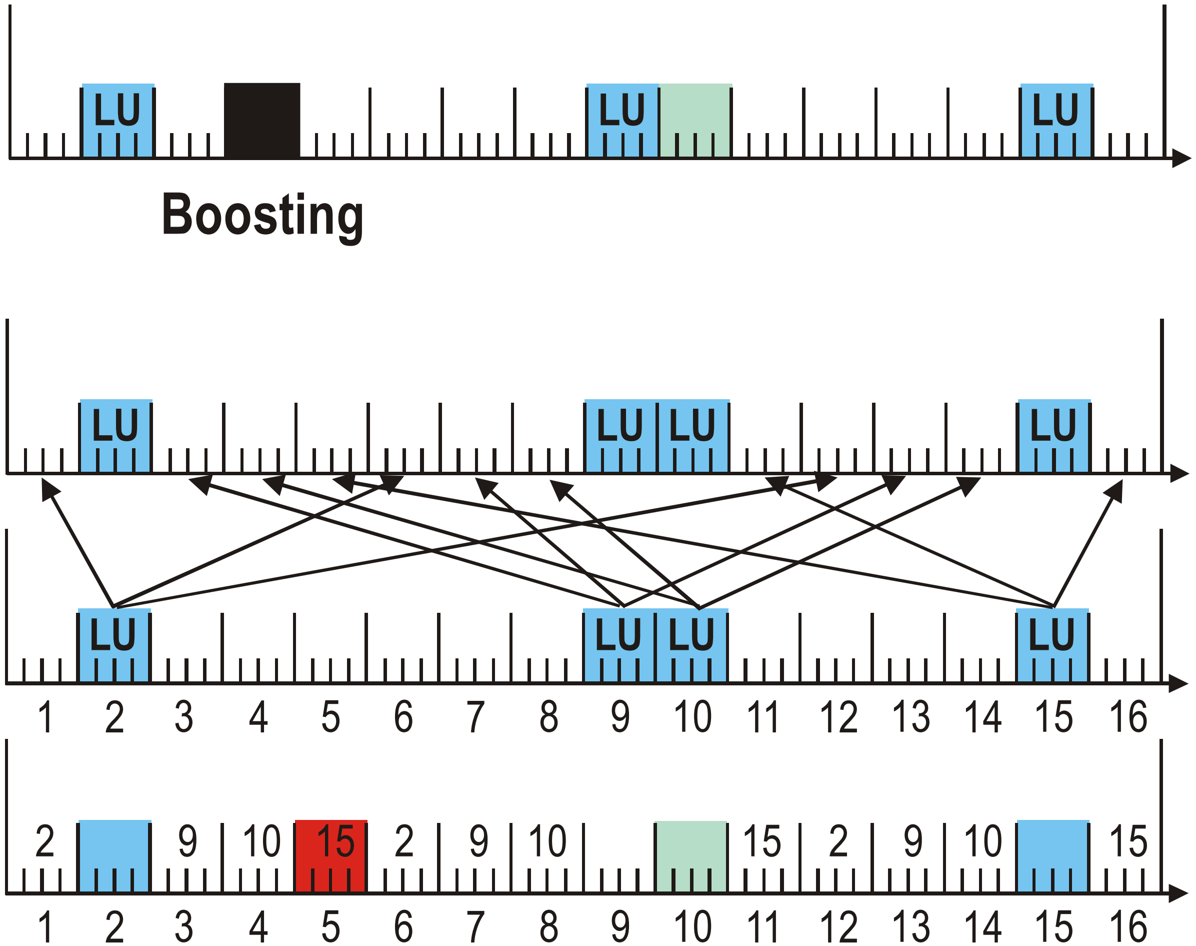
New constellation (after the detection phase) :



BOOSTING PROTOCOL (2)

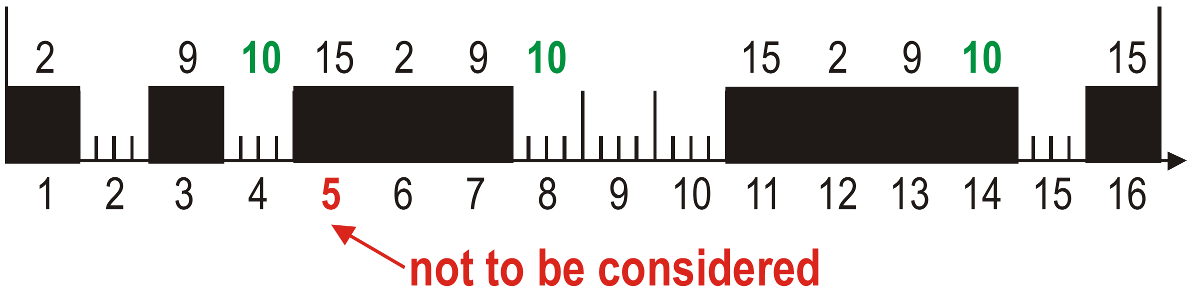
Signaling of *newly occupied* channels

Mapping phase

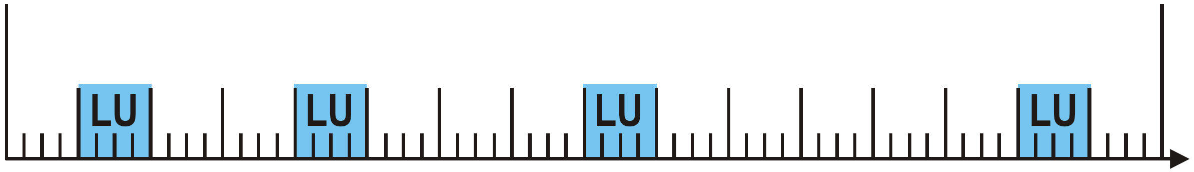


BOOSTING PROTOCOL (3)

Signaling of further occupied LU channels

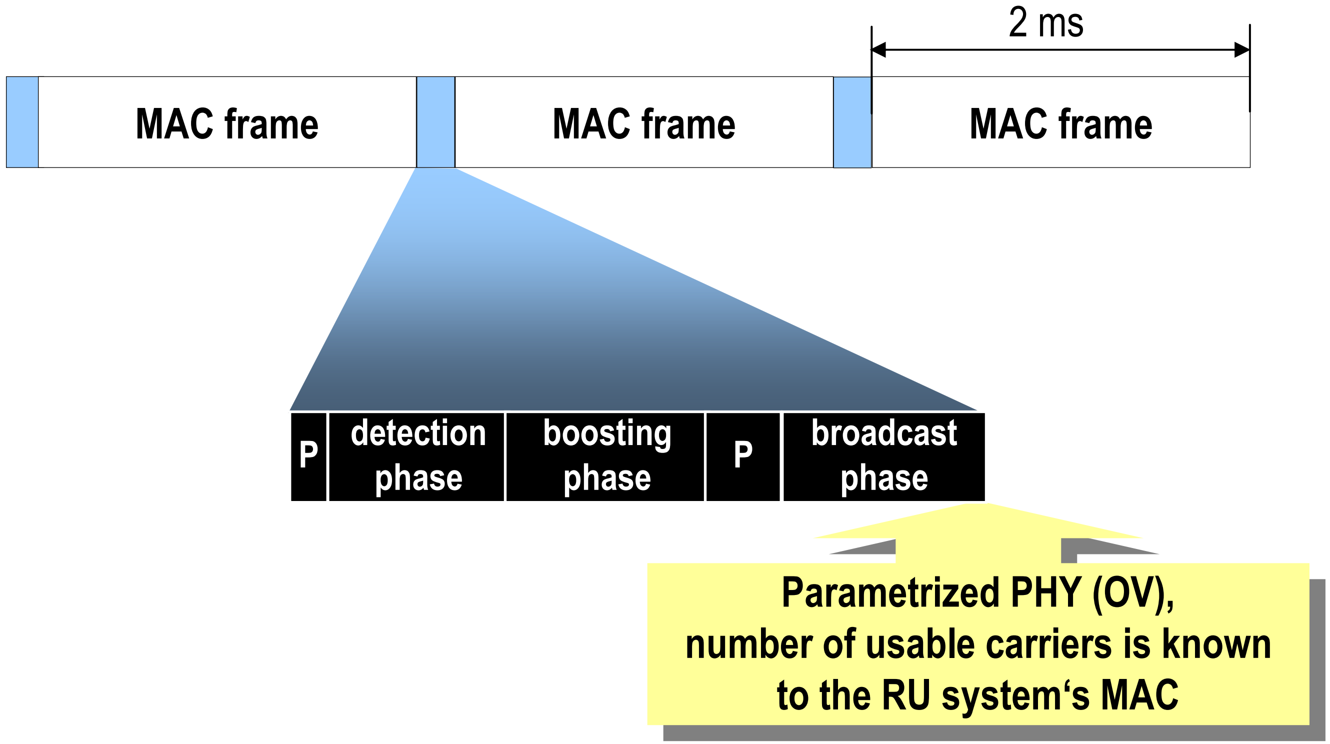


New OV (to be signalized by the AP)



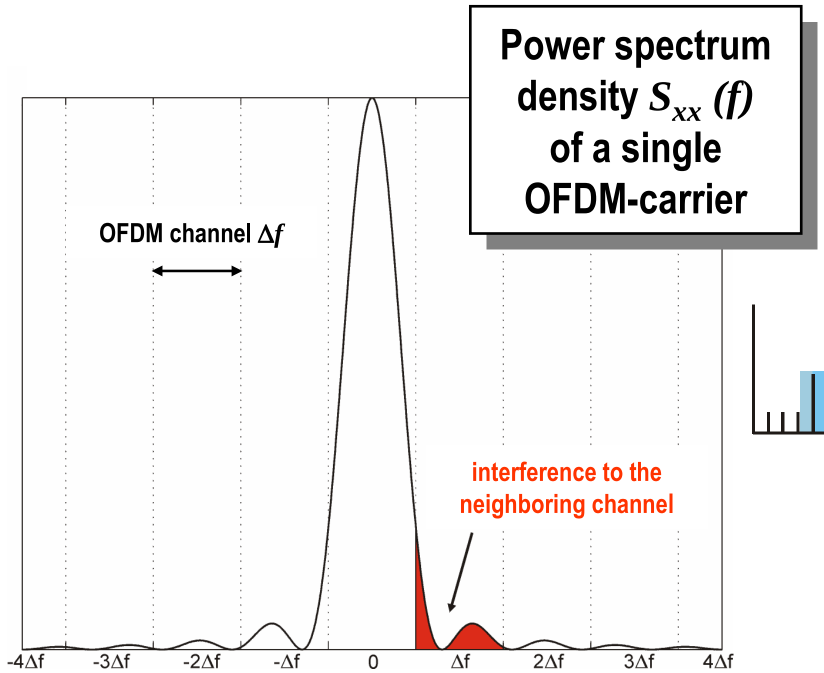
SUMMARY (1)

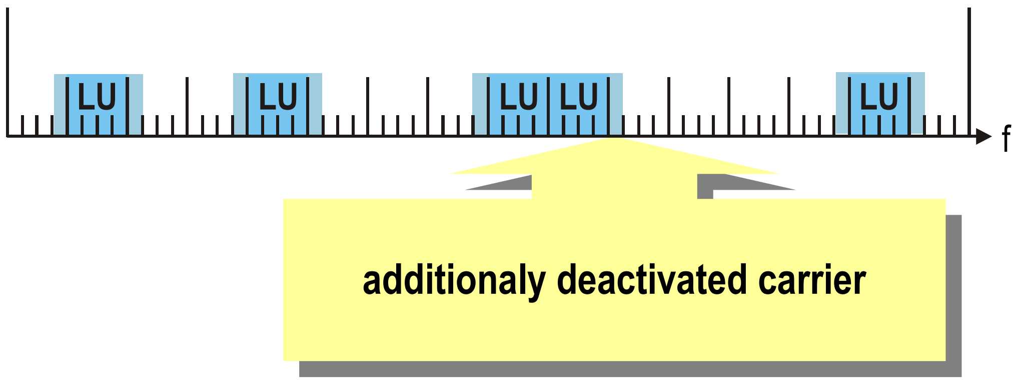
- **Divided detection combined with a boosting protocol and robust Occupancy Vector signaling solves the LU detection problem and leads to a common base of the Physical Layer (PHY).**



SUMMARY (2)

- **The RUs system‘s efficiency is mainly determined by the interference reducing measures!**





OUTLOOK

- Integration of the results into our OMNeT++ software demonstrator.

- Simulation with respect to all effects and with realistic channel models

  $\rightarrow$ Tuning of the free parameters

- Adaptive modulation for optimal use of disturbed channels (FFT leakage)

- MAC layer: Investigation of scheduling algorithms particulary resistent against bandwidth variations

ACKNOWLEDGEMENT

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- The work was mainly done by the INT staff members

  - Dipl.-Ing. Timo Weiß

  - Dipl.-Ing. Fatih Capar

  - Ihan Martoyo, M.Sc.

  and by our graduate students

  - Jörg Hillenbrand

  - Albert Krohn

MULTIDIMENSIONAL GAUSSIAN DENSITY

- **There is No Line of Sight (NLOS) from the LU to be detected to the measuring RU**

$$f_S(\mathbf{z}) = \frac{1}{\sqrt{(2\pi)^{2n} \det \mathbf{C}_{SS}}} \exp\left(-\frac{1}{2}\mathbf{z}^T \mathbf{C}_{SS}^{-1}\mathbf{z}\right)$$

$$\mathbf{C}_{SS} = \mathbf{C}_{\mathbf{ZZ}} = \begin{pmatrix} \mathbf{C}_{\mathbf{XX}} & \mathbf{C}_{\mathbf{XY}} \\ \mathbf{C}_{\mathbf{YX}} & \mathbf{C}_{\mathbf{YY}} \end{pmatrix} = \begin{pmatrix} \mathbf{C}_{\mathbf{XX}} & \mathbf{C}_{\mathbf{XY}} \\ \mathbf{C}_{\mathbf{XY}} & \mathbf{C}_{\mathbf{XX}} \end{pmatrix}$$

$$\mathbf{C}_{XX} = \begin{pmatrix} \sigma_{x_1}^2 & \sigma_{x_1x_2} & \sigma_{x_1x_3} & \cdots & \sigma_{x_1x_n} \\ \sigma_{x_1x_2} & \sigma_{x_1}^2 & \sigma_{x_1x_2} & \cdots & \sigma_{x_2x_n} \\ \sigma_{x_1x_3} & \sigma_{x_1x_2} & \sigma_{x_1}^2 & \cdots & \sigma_{x_3x_n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \sigma_{x_1x_n} & \sigma_{x_2x_n} & \sigma_{x_3x_n} & \cdots & \sigma_{x_1}^2 \end{pmatrix}$$

TOTAL DENSITY

- **Noise component density (AWGN)**

$$f_N(\mathbf{z}) = \frac{1}{\left(2\pi\sigma_N^2\right)^{n}} \exp\left(-\frac{\mathbf{z}^T\mathbf{z}}{2\sigma_N^2}\right)$$

- **Resulting density**
  - **Convolution of the single densities**

$$f_{R|\text{ no LU}}\left(\mathbf{z}|\text{ no LU}\right) = f_N(\mathbf{z})$$

$$f_{R|\text{ LU}}\left(\mathbf{z}|\text{ LU}\right) = \frac{1}{\sqrt{(2\pi)^{2n}\det\left(\mathbf{C}_{SS}+\sigma_N^2\mathbf{E}\right)}} \exp\left(-\frac{1}{2}\mathbf{z}^{\mathbf{T}}\left(\mathbf{C}_{SS}+\sigma_N^2\mathbf{E}\right)^{-1}\mathbf{z}\right)$$

  - **Derivation of an optimal estimator**

ESTIMATOR

- **Neyman-Pearson criterion:** Maximization of the detection probability $\boldsymbol{P_D}$ for a given false alarm probability $\boldsymbol{P_F}$

$$P_F = \int_{\mathbf{R}_1} f_{R|\text{no LU}}(\mathbf{z}|\text{no LU})\,d\mathbf{z}$$

$$P_D = \int_{\mathbf{R}_1} f_{R|\text{LU}}(\mathbf{z}|\text{LU})\,d\mathbf{z}$$

Likelihood ratio:

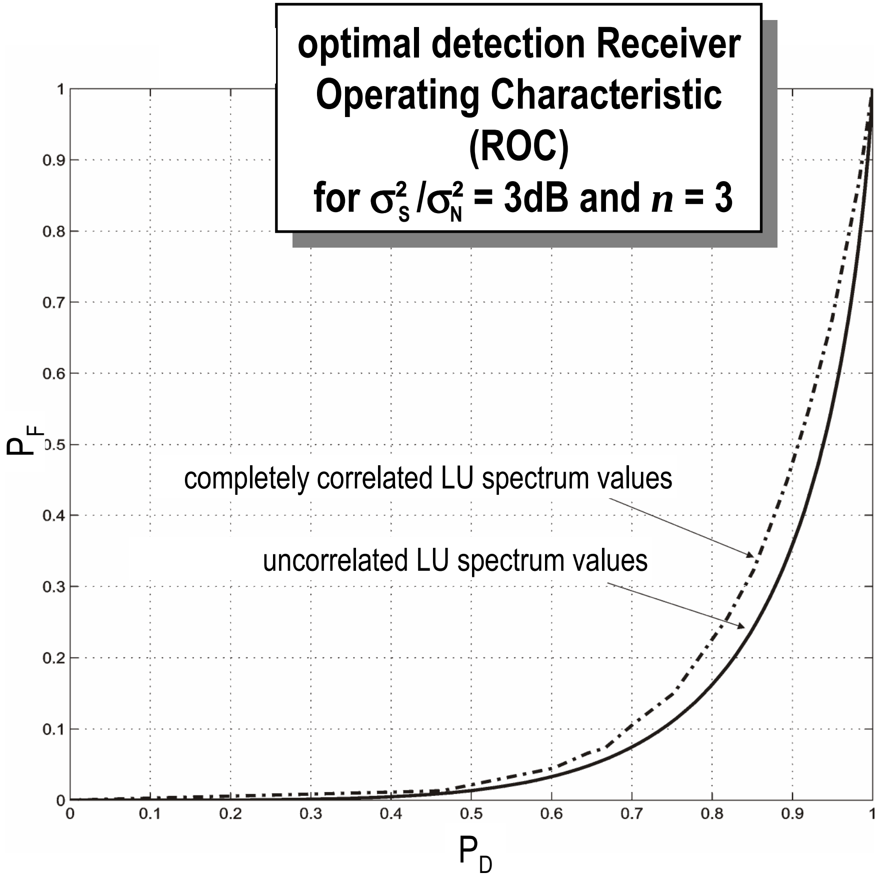
$$\frac{f_{R|\text{LU}}(\mathbf{z}|\text{LU})}{f_{R|\text{no LU}}(\mathbf{z}|\text{no LU})} \overset{\text{LU}}{>} \lambda_0$$

Optimal estimator:

$$\mathbf{z}^T\left(\left(\mathbf{C}_{SS}+\sigma_N^2\mathbf{E}\right)^{-1}-\left(\sigma_N^2\mathbf{E}\right)^{-1}\right)\mathbf{z} \overset{\text{LU}}{<} -2\ln\left(\lambda_0\frac{\sqrt{\det\left(\mathbf{C}_{SS}+\sigma_N^2\mathbf{E}\right)}}{\left(\sigma_N^2\right)^n}\right)$$

UNKNOWN COVARIANCE MATRIX

- **Problem: For the optimal estimator C_{SS} must be known**



$\longrightarrow$ **mismatched estimator!**

- Uncorrelated spectrum values for the LU receiving process:

$$\mathbf{z}^T\mathbf{z} = |\mathbf{z}|^2 \overset{\text{SN}}{>} 2\,\frac{\sigma_S^2+\sigma_N^2}{\sigma_S^2/\sigma_N^2}\left(\ln(\lambda_0) + n\ln\left(\frac{\sigma_S^2}{\sigma_N^2}+1\right)\right)$$

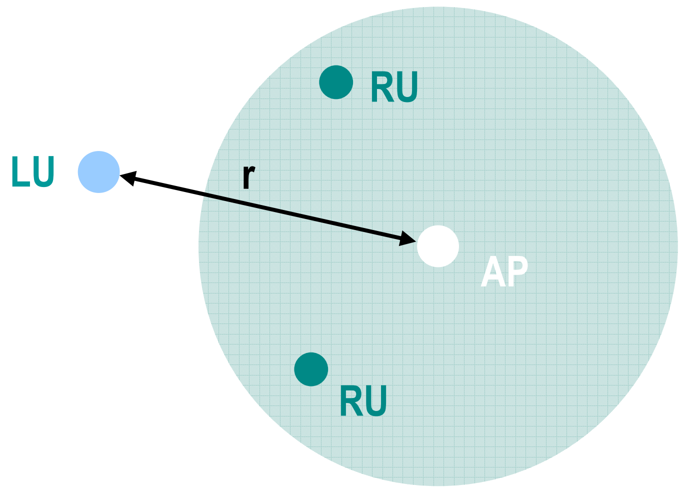
- Completely correlated real parts and imaginary parts of the spectrum values:

$$\left(\sum_{i=1}^{n} x_i\right)^2 + \left(\sum_{i=1}^{n} y_i\right)^2 \overset{\text{SN}}{>} 2\,\frac{n\,\sigma_S^2+\sigma_N^2}{\sigma_S^2/\sigma_S^2}\,\ln\left(\lambda_0\left(n\frac{\sigma_S^2}{\sigma_N^2}+1\right)\right)$$

SNR_D ESTIMATION

- **For which average power of a received LU signal must the LU channel be classified as used?**

$\rightarrow$ depends on the permissible interferences on the LUs



AP *Access Point*

$$SNR_D\,[\mathrm{dB}] \overset{!}{\leq} 10\log\left(10^{\frac{\Delta SNR\,[\mathrm{dB}]}{10}} - 1\right) + \Delta P\,[\mathrm{dB}]$$

DETERMINATION OF SNR_D

ΔP [dB]	ΔSNR [dB]	SNR_D [dB]
0	1	−5.8
0	2	−2.3
0	3	0.0
0	4	1.8
0	5	3.3
3	1	−2.8
3	2	0.7
3	3	3.0
3	4	4.8
3	5	6.3

ΔP [dB]	ΔSNR [dB]	SNR_D [dB]
6	1	0.2
6	2	3.7
6	3	6.0
6	4	7.8
6	5	9.3
10	1	4.2
10	2	7.7
10	3	10.0
10	4	11.8
10	5	13.3

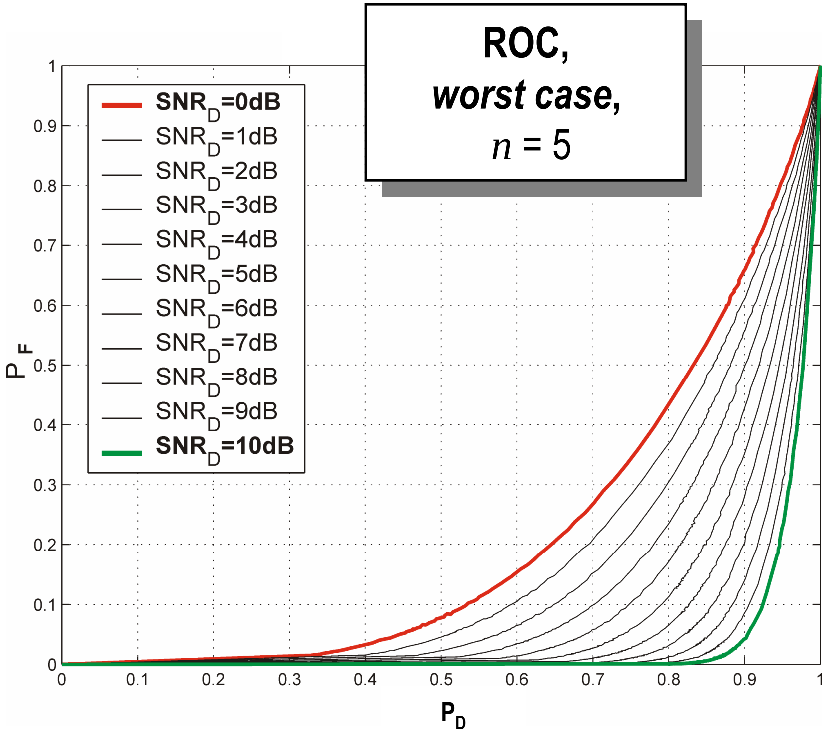
Higher SNR_D ➡ lower P_F ➡ enhanced efficiency

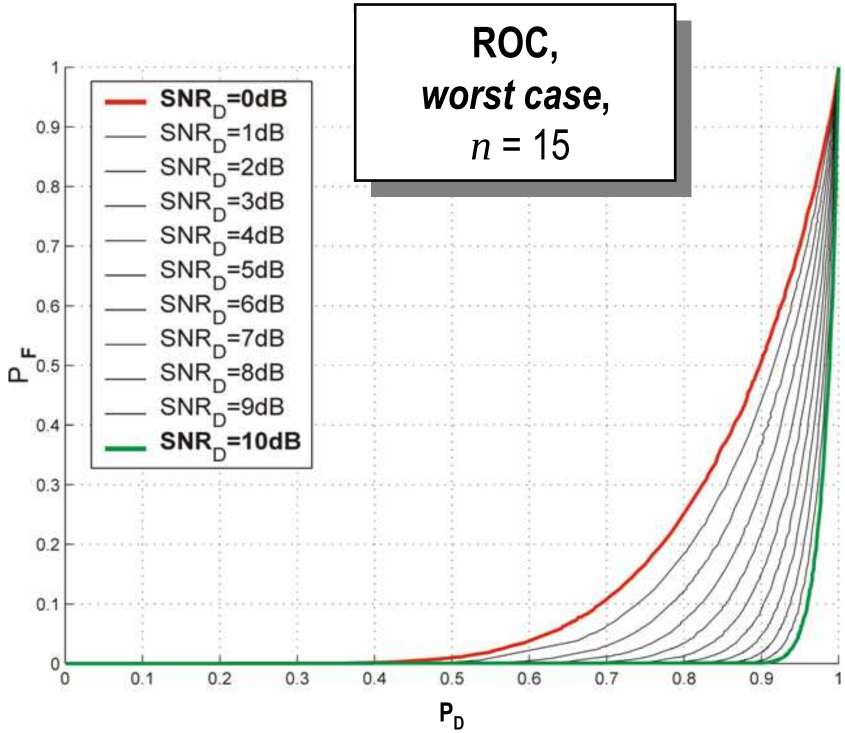
high ΔP is advantageous for detection

OPERATING CHARACTERISTI

- **Simulation results: *worst case* consideration**

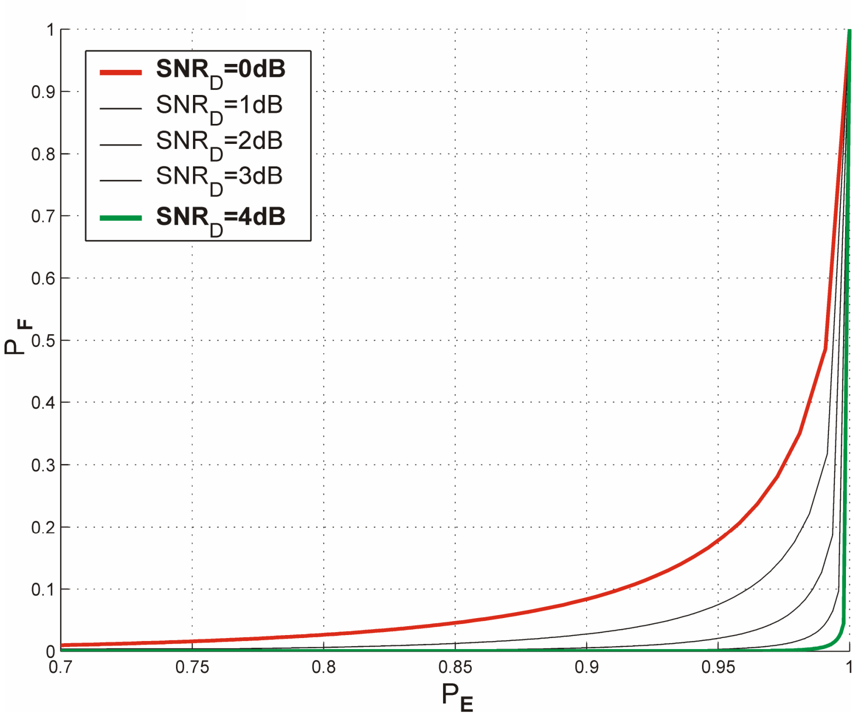
  ↪ maximal mismatched estimator





ROCs

- ***best case* consideration:** Uncorrelated spectrum values



ROC,
***best case*,**
n = 15

Reality is somewhere between *best* und *worst case*
$\rightarrow$ Choose n for the *worst case* !?

DIVERSITY SOLUTION

Given detection probability : $P_D = 0{,}999$

Problems:

multipaths *(fading)*

too high false alarm probability P_F

P_D cannot be realized with only one measurement station!

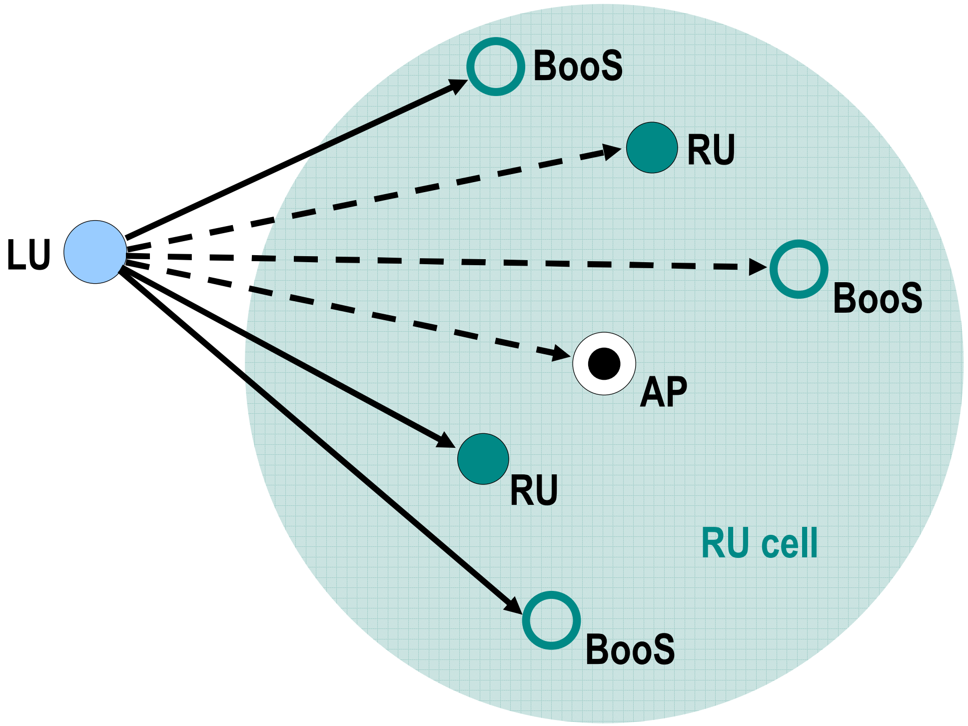
Solution:

distributed detection *(diversity)*

all *mobile terminals* and three additional *Boosting Stations* take measurements jointly

P_D becomes realizable for moderate P_F !

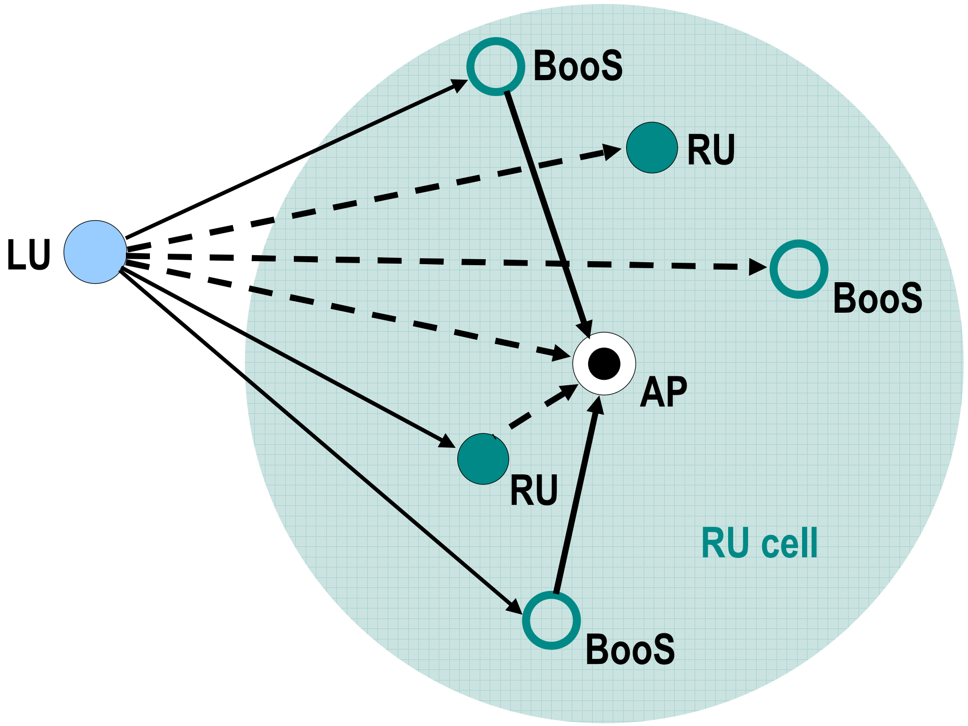
DISTRIBUTED DETECTION



AP	***A**ccess **P**oint*
BooS	***Boo**sting **S**tation*
LU	***L**icensed **U**ser*
RU	***R**ental **U**ser*

SIGNALING (1)

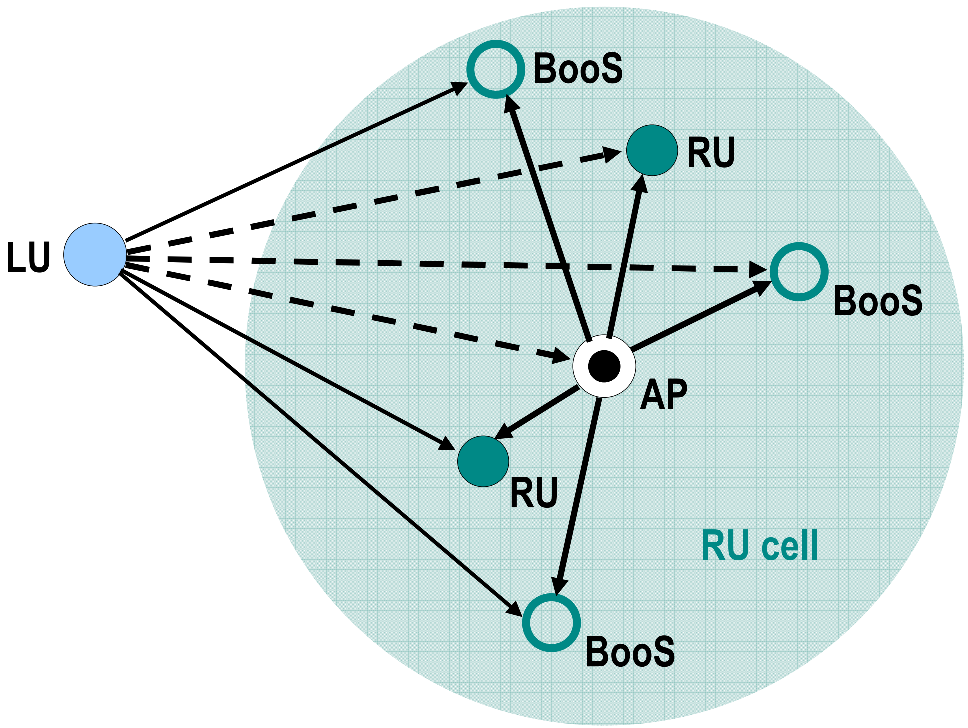
- **RUs + BooSs $\rightarrow$ (AP): *Boosting protocol***



AP computes the elementwise „exclusive or“ of all OVs

SIGNALING (2)

- **AP → RUs + BooSs: *Robust time-frequency broadcast***



AP	***A**ccess **P**oint*
BooS	***Boo**sting **S**tation*
LU	***L**icensed **U**ser*
RU	***R**ental **U**ser*

DIVERSITY-GAIN

What is the gain of divided detection ?	
diversity	➔ no fading
P_D for a specific RU may be reduced	➔ P_F decreases

The individual detection results are statistically independent.
If the receiving conditions at the (m) measuring stations (RUs and BooSs) are similar, we get :

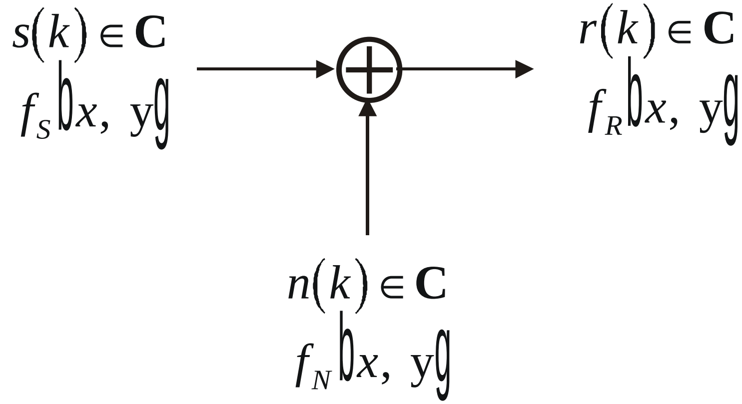
$$P_F^Z(m) \approx 1-(1-P_F)^m$$
$$P_E^Z(m) \approx 1-(1-P_E)^m$$

$$P_F^Z(m) = 1-\left(1-P_F(1-\sqrt[m]{(1-P_E^Z)})\right)^m$$

m	P_D	P_F	P^Z_F
1	0.999	0.982	0.982
2	0.968	0.662	0.886
3	0.900	0.294	0.648
4	0.822	0.100	0.344
10	0.499	0.001	0.010
20	0.292	≈ 0	≈ 0

LU TRANSMISSION MODEL

- **Signal model**



- **Transition to n FFT repetitions**

$$f_S(x, y) \Rightarrow f_s(\mathbf{x}, \mathbf{y}) \qquad \mathbf{R}^2 \rightarrow \mathbf{R}^{2n}$$

$$f_S(\mathbf{x}, \mathbf{y}) = f_S(\underbrace{x_1, x_2, \ldots, x_n}_{n \text{ real parts}}, \underbrace{y_1, y_2, \ldots, y_n}_{n \text{ imaginary parts}}) = f_S(\mathbf{z}) \text{ where } \mathbf{z} = (\mathbf{x}, \mathbf{y})^T$$